Optimal Exchange Rate Policy and Business Cycles[*]

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Abstract

Implementation and collapse of exchange rate pegging schemes are recurrent events. A currency crisis is often followed by an economic downturn while pegging often begets a boom. In this paper I study why a benevolent central bank should pursue a monetary policy that leads to those recurrent currency crises and subsequent periods of pegging. I show that the optimal policy induces a competitive equilibrium that displays a boom in periods of below average devaluation and a recession in periods of above average devaluation. Therefore, a currency crisis (pegging) can be understood as an optimal policy answer to a recession (boom).

Keywords: Exchange Rate Policy, Business Cycles.

JEL Codes: E32, F31, F41.

1. Introduction

In June 1975, a 16% currency devaluation took place in Argentina. Up to the previous month the nominal exchange rate was fixed. The country's GDP, which had grown 8.5% in 1974, fell 2.7% in 1975. A reverse episode happened in 1991. The Argentine currency devalued 68.7% in January and only 5.9% in February. The GDP rose 8.9% in 1991, against the modest increase of 0.1% in 1990.

Instead of being an isolated episode, the Argentine experience illustrates a general pattern. The empirical works of Frankel and Rose (1996), Klein and Marion (1997), and Milesi-Ferretti and Razin (1998) show that several countries have experienced episodes in which pegging schemes were either changed or temporarily abandoned and later reinstated. That is, they show that (i) implementation and collapse of exchange rate pegging schemes are recurrent events. Milesi-Ferretti and Razin (1998) provide evidence that (ii) currency crises are frequently followed by a fall below the trend in output and consumption and a real depreciation of the domestic currency. Kiguel and Liviatan (1992) and Végh (1992) document that (iii) the reverse facts plus a deterioration of the current account and an increase in real wages often accompany a pegging.

This paper is an attempt to shed light on the empirical regularities listed in (i), (ii) and (iii) above. Its first goal is to explain why governments optimally choose to pursue actions that lead to recurrent implementation of pegging policies[1] and their subsequent breakdown. The second goal is to reproduce, at a qualitative level, the corresponding business cycle regularities. To carry out those tasks, I build on Lucas and Stokey's (1983) seminal work on optimal monetary and fiscal policy and study the problem of selecting the optimal monetary and exchange rate policy when the fiscal policy is exogenous.

The environment studied in this paper is an infinite horizon stochastic one. The model is a monetary two sector (tradable and non-tradable) small open economy. Consumers face a cash-in-advance constraint on a fraction of their purchases of non-tradables. Tax rates on labor income, government consumption and a few other variables are stochastic processes.

As often happens in dynamic general equilibrium models, the model considered in this paper is not analytically solvable. Hence, I have to rely on numerical techniques to solve it. As a consequence, the results I summarize next are not necessarily general, in the sense that I cannot be sure that they hold for all conceivable parameter values and stochastic processes. However, I present some sound

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[1]For the purposes of this paper, the expression *pegging policy* does not necessarily mean a fixed exchange rate. I attach this expression to a macroeconomic policy that leads to a rate of depreciation of the domestic currency that is considerably lower than the one that prevailed before the implementation of the policy in question. As an example, consider the Argentine experience described in the first paragraph of this paper. In the end of January of 1991 a crawling peg was introduced and in March of that year a fixed exchange rate policy was adopted. Thus, the pegging policy was implemented in January.

intuition for them; as a consequence, I conjecture that these results are robust ones.

The optimal devaluation rate is a function of the economy's state. As this state changes, the devaluation rate oscillates; that is, the economy shifts from low to high rates of devaluation. These policy switches can happen infinitely many times. I associate them with the implementation and collapse of exchange rate pegging policies.[2]

In periods of fiscal deficits the devaluation rate is higher than in times of surpluses. The intuition is simple. Whenever public expenditures are relatively higher, the optimal policy will prescribe a combination of higher taxation and debt issuance. If the government is not allowed to change the tax rates, the only possible way to raise additional revenue is through inflation. A higher inflation level will determine a higher rate of devaluation of the domestic currency. A positive technological shock that leads to an output rise will reduce the fiscal deficit as a fraction of GDP. The previous reasoning shows that currency devaluation and technological shocks are negatively correlated.

The assumption that fiscal variables are exogenous plays an important role in this paper. In a related paper, Cunha (2008) discussed the problem of the optimal monetary policy in a deterministic version of the model considered in this paper and showed that the well-known Friedman prescription of setting nominal interest rates equal to zero is optimal even if the government can select just a few of all conceivable tax instruments. Thus, for the model to display large changes in the inflation rate and exchange rate devaluation, it is necessary to prevent the government from using tax rates in an optimal fashion. That assumption can be justified on two grounds. First, it usually takes longer for a government to be able change its fiscal policy in reaction to a shock than to change monetary policy. Hence one can interpret the assumption in question as the limiting case where the time required to adjust the fiscal policy goes to infinity. The second factor is that in the real world there is not perfect coordination between fiscal and monetary authorities. Thus, it is important to understand the central bank's reaction to a given fiscal policy. And this is exactly the problem analyzed in this essay.

Let me now consider the business cycle facts associated with a pegging. In response to a shock that decreases the fiscal deficit and increases the productivity, the optimal policy prescribes a decrease in the devaluation rate. Higher productivity leads to higher output. The combination of lower devaluation rate and higher output generates an income effect. People increase their consumption sufficiently to induce a current account deficit. The higher demand for tradable goods can be partially offset by imports. As a consequence, non-tradables become relatively

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[2]I wish to point out that, although the model can account for repeated shifts from low to high devaluation rates, it does not have an outcome in which the exchange rate remains fixed over time. A possible way of changing the model so that the exchange rate will remain fixed in some states consists of assuming, as in Obstfeld (1994), that if the government allows the exchange rate to devaluate, then it will incur a fixed cost.

more expensive. Thus, the real exchange rate appreciates. In similar fashion, a shock that increases the fiscal deficit and reduces productivity leads to a higher devaluation and induces the empirical regularities associated with currency crises.

There is traditional wisdom in which a currency crisis triggers a recession. In this essay, a large devaluation does not cause a recession. The government optimally chooses to devalue the currency at higher rate when the economy hits a bad state. This finding has a striking policy implication. Given that the economy is facing a recession, a devaluation is an optimal answer. Any policy that prevents or just postpones the devaluation will lead to welfare losses.

Exchange rate devaluations are often viewed as a consequence of time consistency problems, as in Obstfeld (1994) and Giavazzi and Pagano (1990). In this essay, devaluations are fully anticipated and are optimal choices for a government that can credibly commit to a policy.

Obstfeld and Rogoff (1995) advocated the use of models with solid micro foundations to study exchange rate policies. Obstfeld (1994) pointed out the relevance of understanding how such a policy is selected. Today, there exist several papers on open economy macro with micro foundations. However, few of them study the selection of the exchange rate policy. This paper attempts to answer the question raised by Obstfeld (1994) using the approach advocated by Obstfeld and Rogoff (1995).

Several studies on currency crises take exchange rate pegging as given and explain why a currency crisis must happen, as in Krugman (1979). Other essays explain why a government chooses to devalue, even if it is still possible to stick to the pegging policy, as in Obstfeld (1994) and Giavazzi and Pagano (1990). Nevertheless, no essay is aimed at explaining why pegging is ever introduced. This essay innovates by adopting a unified framework to explain both pegging episodes and currency crises.

The exchange rate policy is usually assumed to be exogenous in the papers about exchange-rate-based stabilization programs. Rebelo (1997) states that it is important to understand the timing of the stabilization. This paper shows that stabilization may be an optimal answer to a fiscal contraction. This important step is taken in the context of a model that replicates qualitatively several of the stylized facts listed by Mendoza and Uribe (1997), Rebelo (1997), and Rebelo and Végh (1995).

The paper is organized as follows. The model is described in Section 2. Section 3 is devoted to characterization and examples of competitive equilibrium. The problem of selecting policies that lead to the best competitive equilibrium is studied in Section 4, along with the properties of this efficient outcome. Section 5 concludes.

2. The Economy

Consider a small country populated by a continuum of mass one of identical and infinitely lived households and a government. A household is composed of a shopper and a worker, who is endowed with one unit of time.

The country produces two non-tradable goods. The first is consumed by households (c_1^N). The second is consumed by households (c_2^N) and government (g^N). The country also produces a tradable good, which is consumed by households (c^T) and a government (g^T). This last good can also be exported (x) or imported ($-x$).

Markets operate in a particular way. At a first stage of each date t, a spot market for goods and labor services operates. At a second stage, after the market for goods and labor services closes, a security and currency market operates.[3]

A domestic currency M circulates in this economy. Two types of securities are traded: a claim B to one unit of M and a claim B^* to one unit of some foreign currency. Both claims have maturity of one period. Foreigners do not sell or buy claims to the domestic currency. Government and residents can purchase and/or sell the claims B^* at an exogenous price, in terms of the foreign currency, q_t^*.

Workers cannot sell their services outside the country. Shoppers face a cash-in-advance constraint. The purchases of c_1^N must be paid for with the domestic currency. Except for the purchases of that good, all other transactions are settled during the security and currency trading session. The date t price, in terms of the foreign currency, of the tradable good is exogenous and equal to p_t^*.

The technology is described by $0 \leq y^T \leq \theta^T (l^T)^{\alpha^T}$ and $0 \leq y^N \leq \theta^N (l^N)^{\alpha^N}$, where y^T is the tradable output, θ^T a productivity shock and l^T is the amount of labor allocated to the production of that good. Similar meanings are assigned to y^N, θ^N and l^N. Both α^T and α^N lie in the set $(0,1]$.

At each date t labor income is taxed at a proportional rate τ_t. Let s_t denote the vector $(\theta_t^T, \theta_t^N, g_t^T, g_t^N, \tau_t, p_t^*, q_t^*)$. The sequence $\{s_t\}_{t=0}^{\infty}$ is a stochastic process. Each s_t has a support contained in a finite set S. The object s^t stands for a history $(s_0, ..., s_t)$ of events and $s^\infty = (s_0, s_1, ...)$. For a given t, S^t denotes the set of all possible histories s^t and S^∞ is the set of all possible s^∞. For a given s^t in S^t, $\mu(s^t)$ denotes the probability that this particular s^t will be realized. The realization of s_t is known at the beginning of date t. If $k \leq t$, $\mu(s^t|s^k)$ denotes the conditional probability of s^t given s^k; $S^t(s^k)$ is the set of all $s^t \in S^t$ such that the first k events in s^t are equal to s^k. In other words, $S^t(s^k)$ is the set of all possible continuations of the history s^k up to date t.

Each good is produced by a single competitive firm. Let $l(s^t)$ denote the amount of labor supplied by each household at date t if the history s^t occurs. Other variables indexed by s^t have analogous meaning. Feasibility requires

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[3]In this setup, unexpected inflation does not act as a lump sum tax. Therefore, the problem of selecting an optimal policy will have a well defined solution even if the government has some outstanding debt at date zero. See Nicolini (1998) for further details.

$$l^T(s^t) + l^N(s^t) = l(s^t) \leq 1 \ , \ c_1^N(s^t) + c_2^N(s^t) + g_t^N = \theta_t^N [l^N(s^t)]^{\alpha^N} \ ,$$

$$c^T(s^t) + g_t^T + x(s^t) = \theta_t^T [l^T(s^t)]^{\alpha^T} \ . \tag{1}$$

The government finances the sequence $\{g_t^T, g_t^N\}_{t=0}^\infty$ by issuing and withdrawing the domestic currency, by issuing and redeeming claims B, by purchasing and selling B^*, by taxing labor income, and by levying lump-sum taxes on profits.[4] Its budget constraint is

$$E(s^t)p_t^* g_t^T + p^N(s^t)g_t^N + B(s^{t-1}) + E(s^t)q_t^* B_G^*(s^t) + M(s^{t-1}) =$$

$$\tau_t w(s^t)l(s^t) + q(s^t)B(s^t) + E(s^t)B_G^*(s^{t-1}) + M(s^t) + \psi^T(s^t) + \psi^N(s^t) \ , \tag{2}$$

where $p^N(s^t)$, $w(s^t)$ and $q(s^t)$ are the respective date t monetary prices (in terms of the domestic currency) of the non-tradable good, labor services and the domestic claim; $E(s^t)$ is the nominal exchange rate; $B_G^*(s^t)$ stands for the foreign assets held by the government at the end of date t; $M(s^t)$ and $B(s^t)$ are the amounts of domestic currency and public debt held by the households at the end of date t; $\psi^T(s^t)$ and $\psi^N(s^t)$ are the date t profits. All those variables are conditional on the history of events. A negative value for $B_G^*(s^t)$ means that the government is borrowing abroad, while a negative value for $B(s^t)$ means that the government is lending to domestic residents. At $t = 0$ the government holds an initial amounts $\bar{B}_G^*$ of foreign assets. To avoid Ponzi schemes, a standard boundedness constraint $|B_G^*(s^t)/p_t^*| \leq A < \infty$ is imposed on government foreign assets.

The function $u : \mathbb{R}_+^3 \times [0,1] \to \mathbb{R} \cup \{-\infty\}$,

$$u(c^T, c_1^N, c_2^N, l) = \frac{\left[(c^T)^{\gamma^T}(c_1^N)^{\gamma_1}(c_2^N)^{\gamma_2}(1-l)^{\gamma^l}\right]^{1-\sigma}}{1-\sigma} \ , \tag{3}$$

is the typical household period utility function. Each γ is positive and they add up to 1, while $\sigma \geq 0$. Intertemporal preferences are described by

$$\sum_{t=0}^\infty \sum_{s^t \in S^t} \beta^t \mu(s^t) u\left(c^T(s^t), c_1^N(s^t), c_2^N(s^t), l(s^t)\right) \ , \tag{4}$$

where $\beta \in (0,1)$. The date t budget constraint of the typical household is

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[4]It is usually assumed that profits are appropriated by households. Sticking to that procedure would make the forthcoming implementability constraint (8) depend on government consumption. That would make the computation of the optimal policies more expensive. Thus, for simplicity I assume that government taxes profits away. This assumption is not essential to the results.

$$E(s^t)p_t^* c^T(s^t) + p^N(s^t)[c_1^N(s^t) + c_2^N(s^t)] + q(s^t)B(s^t) + E(s^t)q_t^* B_H^*(s^t) + M(s^t) \leq$$

$$(1 - \tau_t)w(s^t)l(s^t) + B(s^{t-1}) + E(s^t)B_H^*(s^{t-1}) + M(s^{t-1}) , \qquad (5)$$

where $B_H^*(s^t)$ stands for the foreign assets held by the household at the end of date t if history s^t occurs. The constraints $|B(s^t)/p^N(s^t)| \leq A$ and $|B_H^*(s^t)/p_t^*| \leq A$ prevent Ponzi games. People face the cash-in-advance constraint

$$p^N(s^t)c_1^N(s^t) \leq M(s^{t-1}) . \qquad (6)$$

Given initial cash and bond holdings $(\bar{M}, \bar{B}, \bar{B}_H^*)$, a household chooses a state contingent sequence $\{[c^T(s^t), c_1^N(s^t), c_2^N(s^t), l(s^t), B(s^t), B_H^*(s^t)]_{s^t \in S^t}\}_{t=0}^\infty$ to maximize (4) subject to the constraints (5), (6), and $l(s^t) \leq 1$. Except for $B(s^t)$ and $B_H^*(s^t)$, all those variables are constrained to be non-negative.

3. Competitive Equilibrium

A history contingent date t price vector $(E(s^t), p^N(s^t), w(s^t), q(s^t))$ is denoted by $\varphi(s^t)$. Date t history contingent asset holdings vector $(M(s^t), B(s^t), B_H^*(s^t), B_G^*(s^t))$ is denoted by $\zeta(s^t)$, while $\chi(s^t)$ denotes the vector $(l^T(s^t), l^N(s^t), x(s^t), c^T(s^t), c_1^N(s^t), c_2^N(s^t), l(s^t))$. Additionally, $\varphi = \{[\varphi(s^t)]_{s^t \in S^t}\}_{t=0}^\infty$, $\chi = \{[\chi(s^t)]_{s^t \in S^t}\}_{t=0}^\infty$ and $\zeta = \{[\zeta(s^t)]_{s^t \in S^t}\}_{t=0}^\infty$.

A *competitive equilibrium* is an object (φ, χ, ζ) satisfying: (i) given φ, (χ, ζ) provides a solution for the household problem; (ii) given $\varphi(s^t)$, $l^T(s^t)$ and $l^N(s^t)$ maximize the respective firm's profit; (iii) (1) and (2) hold.

Two points in the above definition should be emphasized. Item (ii) is equivalent to $w(s^t) = p^N(s^t)\alpha^N \theta_t^N [l^N(s^t)]^{\alpha^N - 1}$ and $w(s^t) = E(s^t)p_t^* \alpha^T \theta_t^T [l^T(s^t)]^{\alpha^T - 1}$. Adding the identities $\psi^N(s^t) + w(s^t)l^N(s^t) = p^N(s^t)[\, c_1^N(s^t) + c_2^N(s^t) + g_t^N]$ and $\psi^T(s^t) + w(s^t)l^T(s^t) = E(s^t)p_t^*[c^T(s^t) + g_t^T + x(s^t)]$ to (2) and (5) taken as equality, one obtains

$$p_t^* x(s^t) + B_G^*(s^{t-1}) + B_H^*(s^{t-1}) - q_t^* B_G^*(s^t) - q_t^* B_H^*(s^t) = 0 , \qquad (7)$$

which is the balance-of-payments identity for this economy. So, it is not necessary to spell this condition out when defining competitive equilibrium.

I want to obtain a characterization of the set of all competitive equilibrium allocations. As usual, although simple, that exercise is a long one. The techniques presented in Lucas and Stokey (1983) and Chari and Kehoe (1999) are the starting point. This task is carried out, with all details, for a deterministic version of the economy considered in this paper in the working paper version of Cunha (2008). Since the generalization for the stochastic case is neither difficult nor time consuming, I refer the interested reader to that paper.[5]

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[5] Available at http://ideas.repec.org/p/ibr/dpaper/2005-06.html.

To simplify the notation, $u(s^t)$, $u_T(s^t)$, $u_1(s^t)$, $u_2(s^t)$, and $u_l(s^t)$ will denote, respectively, the value of u and its partial derivatives $\partial u/\partial c^T$, $\partial u/\partial c_1^N$, $\partial u/\partial c_2^N$, and $\partial u/\partial l$ evaluated at the point $(c^T(s^t), c_1^N(s^t), c_2^N(s^t), l(s^t))$. The sum $u_T(s^t)c^T(s^t) + u_1(s^t)c_1^N(s^t) + u_2(s^t)c_2^N(s^t) + u_l(s^t)l(s^t)$ will be denoted by $W(s^t)$.

There are six conditions with obvious economic meaning that must hold in any competitive equilibrium. A trivial condition is (1). The second is

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \mu(s^t) W(s^t) = u_1(s^0)c_1^N(s^0) + \tag{8}$$

$$u_2(s^0)\left[\frac{\bar{B}}{p^N(s^0)} + \frac{\bar{M}}{p^N(s^0)} - c_1^N(s^0)\right] + u_T(s^0)\frac{\bar{B}_H^*}{p_0^*} \ ,$$

which consolidates all date t budget constraints of the households. The third is a balance-of-payment constraint

$$-\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \mu(s^t) u_T(s^t) x(s^t) = u_T(s^0)\frac{\bar{B}_H^* + \bar{B}_G^*}{p_0^*} \ , \tag{9}$$

which requires imports to be financed by the country's initial wealth. The fourth requirement, ensuring that people's marginal rate of substitution is consistent with the q_t^* and p_t^*, is

$$q_t^* \frac{\mu(s^t)u_T(s^t)}{p_t^*} = \beta \sum_{s_{t+1} \in S} \frac{\mu(s^t, s_{t+1})u_T(s^t, s_{t+1})}{p_{t+1}^*} \ . \tag{10}$$

The fifth constraint is that households' marginal rate of substitution between tradables and non-tradables must match the marginal rate of transformation between those types of goods, i.e.,

$$\frac{u_T(s^t)}{u_2(s^t)} = \frac{\alpha^N \theta_t^N [l^T(s^t)]^{1-\alpha^T}}{\alpha^T \theta_t^T [l^N(s^t)]^{1-\alpha^N}} \ . \tag{11}$$

This equation is also an implementability condition for the real exchange rate $\frac{E(s^t)p_t^*}{p^N(s^t)}$. The sixth,

$$(1 - \tau_t)\frac{\alpha^N \theta_t^N}{[l^N(s^t)]^{1-\alpha^N}} = -\frac{u_l(s^t)}{u_2(s^t)} \ , \tag{12}$$

is an implementability constraint for labor income taxation.

The above constraints are not enough to characterize a competitive equilibrium. Other conditions have to be imposed. The inequalities

$$p^N(s^0)c_1^N(s^0) \leq \bar{M} \tag{13}$$

and

$$u_2(s^t) \leq u_1(s^t) \tag{14}$$

ensure that cash-in-advance constraints hold, while

$$\lim_{t \to \infty} \beta^t \sum_{s^t \in S^t(s^k)} \mu(s^t) u_1(s^t) c_1^N(s^t) = 0 \tag{15}$$

is an implementability constraint for a transversality condition.

The above set of constraints does not include an implementability condition for the government budget constraint. However, there are implementability conditions for people's budget constraint, resource constraints and balance-of-payments. The government budget constraint is a linear combination of those other constraints.

Although fiscal variables are exogenous, the government can pursue several distinct policies. To clarify this point, consider the simple case in which the government has no source of revenue but inflation, no net debt at date zero and its consumption is always positive. The government can balance its lifetime budget with a constant inflation rate and borrow abroad to finance temporary imbalances. It is also possible to balance the budget period by period solely with inflation tax. In this case the inflation does not need to be constant. Different policies will induce distinct competitive equilibria.

4. Optimal Policy

An efficient policy is one that induces the highest attainable value for (4). Recall that $\{g_t^T, g_t^N, \tau_t\}_{t=0}^{\infty}$ is exogenous. Thus, the planner's problem consists of choosing paths for money supply, domestic debt and external borrowing to maximize people's welfare. One can see this as the problem faced by a benevolent central bank that takes the fiscal policy as given.

As pointed out by Chari and Kehoe (1990, 1999) in similar contexts, to characterize the optimal policy and its corresponding outcome, it suffices to find an array $(p^N(s^0), \chi)$ that solves the problem of maximizing (4) subject to (1) and (8)-(15). Following those authors, I say that a solution of this problem is a *Ramsey outcome*.

Next I consider several examples that illustrate the properties of the Ramsey outcome. In the Appendix I provide a sketch of the solution of each of example. Before proceeding, it is convenient to outline the approach adopted. For didactical reasons, I start studying the properties of the optimal policy in a very simple setup: a one-sector closed-economy deterministic version of the model with logarithmic preferences. After analyzing this simple problem, I will study open economies, which constitute my focus in this paper. The reader will be able to observe that each example allows understanding different properties of the efficient outcome and how those properties relate to the economic structure and the parameter values.

Example 1 *Consider a standard closed economy with cash and credit goods. The technology is described by $c_{1t} + c_{2t} + g_t = l_t$. The period utility function is $u =$*

$\log c_1 + \log c_2 + \log(1 - l)$. *Labor income is not taxed, i.e. $\tau_t = 0$, and the initial value of the public debt is zero. The optimal allocation is stationary, in the sense that if $g_t = g_k$, then $(c_{1t}, c_{2t}) = (c_{1k}, c_{2k})$. Assume that $g_t = \bar{g}$ if t is odd and $g_t = g$ if t is even, with $\bar{g} > g$. Let (c_1, c_2) and $(\bar{c}_1, \bar{c}_2)$ denote the corresponding optimal consumption values. Denote the inflation rate $(p_{t+1} - p_t)/p_t$ by π_{t+1}. From the household's first order condition, $1 + \pi_{t+1} = \beta c_{2t}/c_{1t+1}$. Thus, $\pi_t = \bar{\pi} = \beta \bar{c}_2/\bar{c}_1 - 1$ for t odd and $\pi_t = \pi = \beta c_2/c_1 - 1$ for t even, with $\bar{\pi} > \pi$. As shown in the Appendix, $c_1 > \bar{c}_1$ and $c_2 > \bar{c}_2$. Hence, $\bar{c}_2/c_1 < c_2/c_1 < c_2/\bar{c}_1$. Therefore, the inflation rate alternates between a high and a low levels.*

In the above example, the inflation rate is higher in periods of higher public expenditures, although there are attainable policies that lead to constant inflation. Despite the oscillation in the inflation rate, the government does not balance the budget solely with inflation tax, because the real public debt is not constant (the behavior of the public debt can be checked by a recursive evaluation of the household's lifetime budget constraint).

The intuition for this finding is simple. Lucas and Stokey (1983) consider similar models in which the government can choose τ. Although the public debt is used to smooth taxation out over time, τ is not constant. In times of high public expenditures the optimal policies require high tax rates. In the two examples above, inflation is the only taxation instrument available. For this reason, the inflation rate is higher in the states with higher g and lower in the other states.

I now turn to the study of the optimal policy in an open economy. As a first step, just open the one-sector economy of Example 4. One could expect inflation and exchange rate devaluation to be high in times of fiscal deficit and low in times of fiscal surplus. However, that will not occur. Foreign borrowing and lending provide the Ramsey planner additional possibilities for smoothing taxation out and inflation and devaluation rates will be constant over time. Observe that in such a context, the feasibility constraint becomes $c_{1t} + c_{2t} + x_t + g_t = l_t$. Therefore, the marginal rate of transformation between x_t and g_t is equal to one. Thus, oscillations in g_t can be insured through external borrowing and lending at a constant opportunity cost. That does not happen in a two-sector model. Consider the last two equalities in (1). A shock in g_t^T can be offset by a change in $x(s^t)$ at a constant opportunity cost. However, that is not true for a shock in g_t^N. Thus, the optimal policy will prescribe changes in the inflation and devaluation rates only if there are shocks that hit the non-tradable sector.

I next consider two-sector stochastic economies. In Examples 2 and 3, $\{\theta_t^N, g_t^N, \tau_t\}_{t=0}^\infty$ is a Markov process with state space $\{a, b\}$, where $a = (\theta_a^N, g_a^N, \tau_a)$ and $b = (\theta_b^N, g_b^N, \tau_b)$. The transition probabilities are $\mu_{ab} = 0.4$ and $\mu_{ba} = 0.7$. The period utility function is $u = \log c^T + \log c_1^N + \log c_2^N + \log(1 - l)$. Technology is described by $c_1^N(s^t) + c_2^N(s^t) + g_t^N = \theta_t^N l^N(s^t)$ and $c^T(s^t) + x(s^t) = \sqrt{l^T(s^t)}$. The state space is example specific; at date zero the economy is at state b (the bad state in both examples). Finally, $p_t^* = 1$ and $q_t^* = \beta = 0.98$.

Example 2 *In this example, $\theta_a^N = \theta_b^N = 1$, $(g_a^N, \tau_a) = (0.2, 0.3)$, and $(g_b^N, \tau_b) = (0.4, 0.15)$. The nominal exchange rate depreciates 2.2% at state a, which is the state with fiscal surplus. At state b there is a fiscal deficit and the domestic currency devalues 17.8%. As in the previous example, the devaluation rate is larger in periods of fiscal deficit and the government does not rely solely on inflation tax to finance temporary fiscal imbalances.*

A reduction in the fiscal deficit has strong effects on the optimal devaluation rate. The intuition for that is the same provided for Example 1. Let me now discuss other features of the optimal policy and allocation. Total consumption and output are negatively correlated. Therefore, it seems that an economy driven only by fiscal shocks cannot reproduce the empirical regularities mentioned in the Introduction. That observation motivates the next example.

Example 3 *The states are given by $\theta_a^N = 1.25$, $\theta_b^N = 0.75$, and $(g_a^N, \tau_a) = (g_b^N, \tau_b) = (0.25, 0.15)$. The optimal policy prescribes a nominal devaluation of 19.0% at a and 23.4% at b.*

Both output and consumption of non-tradables are negatively correlated with the devaluation rate. However, even an implausibly high dispersion for the technological shocks does not induce large oscillations in the devaluation rate.

In both Examples 2 and 3 the trade balance behaves in a cyclical fashion. This contradicts the empirical evidence that a deterioration of the current account often follows the introduction of a pegging policy. This point will be further discussed after the next two examples.

At this point it is convenient to summarize the findings. The model explains the successive shifts between periods of low and high devaluation rate. Large oscillations in the devaluation rate combined with reasonable oscillations in output require fiscal shocks. On the other hand, fiscal deficits and surpluses alone cannot account for the behavior of output and consumption. These findings will remain true in richer environments. Several unreported experiments corroborate this last assertion.

It is beyond the scope of this essay to explain why technological and fiscal shocks should be positively correlated. A possible explanation is that the inflation tax is a burden not only for consumers, but also for firms. In a more realistic model, a drop in the devaluation rate could induce supply side effects similar to the technological shocks considered here.

So far, I have shown that the model can account for the recurrent shifts from a regime of low devaluation to one of high devaluation. It remains to be shown that the model can also reproduce the stylized facts of currency crises and peggings. The next two examples will reproduce these last empirical regularities.

Before proceeding, I need to explain the model's national accounts. Let e, c and y denote, respectively, real exchange rate, consumption and output. These

variables are quantified according to $e = Ep^*/p^N = u_T/u_2$ (the second equality comes from households' first order conditions), $c = c_1^N + c_2^N + ec^T$ and $y = y^N + ey^T = \theta^N(l^N)^{\alpha^N} + e\theta^T(l^T)^{\alpha^T}$. Sectorial real wages can be evaluated by marginal products. The term fiscal deficit refers to the primary deficit, which is the difference between expenditures $g^N + eg^T$ and fiscal revenue $\tau\theta^N(l^N)^{\alpha^N-1}l + (1-\alpha^N)\theta^N(l^N)^{\alpha^N} + e(1-\alpha^T)\theta^T(l^T)^{\alpha^T}$. In this sum, the first term is the labor income tax revenue and the other two correspond to profits.

Examples 4 and 5 share several features. The environment is deterministic. The period utility is $u = (c^T)^{\gamma^T}(c_1^N)^{\gamma_1}(c_2^N)^{\gamma_2}(1-l)^{\gamma^l}$. Technology is given by $c_{1t}^N + c_{2t}^N + g_t^N = \theta_t^N(l_t^N)^{\alpha^N}$ and $c_t^T + x_t = \theta_t^T(l_t^T)^{\alpha^T}$. As before, $q_t^* = \beta$, $p_t^* = 1$, and $\bar{B} = \bar{B}_H^* = \bar{B}_G^* = 0$. Time is measured in months.

There exists a large body of quantitative literature on the real effects of exchange rate pegging. Preferences and technology will be parameterized to make the economy in this paper resemble some of the models considered in that literature.

Labor income shares $\alpha^T = 0.48$ and $\alpha^N = 0.63$ are borrowed from Rebelo (1997) and Rebelo and Végh (1995). They adopted a quarterly value of 0.99 for β. Converting this figure to monthly units, one obtains $\beta = 0.99^{1/3}$, which is the value adopted in the incoming examples. The share parameter γ^l was set equal to $2/3$, as in Kydland and Prescott (1982). This last values implies $\gamma^T + \gamma_1 + \gamma_2 = 1/3$. In Rebelo (1997) and Rebelo and Végh (1995), tradables and non-tradables have the same share. Thus, $\gamma^T = \gamma_1 + \gamma_2$. The condition $\gamma_1/\gamma_2 = 2/3$ was imposed arbitrarily on those shares. Solving those three equations, one obtains $\gamma^T = 5/30$, $\gamma_1 = 2/30$ and $\gamma_2 = 3/30$.

The constraint $\gamma_1/\gamma_2 = 2/3$ implies that in a steady state with low inflation around 40% of the household expenditures with non-tradables will be paid cash. This is a relatively high number. Hence, the adopted parameterization leads to a cash demand larger than one should expect. Given the fiscal policy, a large money demand will lead to smaller inflation rates. So, this parameterization reduces the model's ability to generate high devaluation rates.

Example 4 *This example reproduces the stabilization stylized facts listed in the Introduction. The economy starts with a loose fiscal policy and low productivity. After five years there is a positive technological shock and a decrease in the fiscal deficit. The behavior of fiscal variables and technology is given by*

$$(g_t^N, \tau_t, \theta_t^N, \theta_t^T) = \begin{cases} (0.31, 0.10, 0.93, 0.93) & \text{if } t < 60 \\ (0.30, 0.22, 1.015, 1.015) & \text{if } t \geq 60 \end{cases}.$$

Although τ has more than doubled, that does not mean that the fiscal tightening is as large as that. Recall that the fiscal revenue is given by labor income taxes plus profits. The former is always between 10% and 25% of the total government revenue. The total fiscal revenue increases by less than 16%. The domestic currency

depreciates 12.7% in nominal terms every month prior to the stabilization and 1.2% after it. Furthermore, x decreases. Both l^N and l^T fall, while both θ^N and θ^T increase. Therefore, the real wage increases in both sectors. The real exchange rate appreciates 1.5%. The intertemporal comparison of consumption and output can be done at with three different choices of e: the one prior to stabilization, the one prevailing after the stabilization, and the current ones. Regardless of the real exchange rate used, both c and y increase. Adopting the e that prevails before the stabilization, output, consumption and fiscal revenue grow, respectively, 1.3%, 9.1% and 15.7%, while government expenditures fall 3.2%.

Example 5 *This example reproduces the currency crisis empirical regularities mentioned in the Introduction. Fiscal variables and technological shocks follow*

$$(g_t^N, \tau_t, \theta_t^N, \theta_t^T) = \begin{cases} (0.26, 0.28, 1.05, 1.05) & \text{if } t < 60 \\ (0.28, 0.10, 0.93, 0.93) & \text{if } t \geq 60 \end{cases}.$$

Hence, during the first five years fiscal policy is tighter and productivity is high. The nominal exchange rate devalues only 0.7% every month. At the beginning of the sixth year, a permanent shock reduces the productivity and increases the fiscal deficit. Government expenditures grow and fiscal revenues fall. The new nominal rate of devaluation of the currency is 19.5% monthly. The real exchange rate depreciates 3.64%. Consumption and output fall, respectively, 10.8% and 0.1% when evaluated with the pre-crisis real exchange rate, while government expenditures grow 7.7% and the fiscal revenue falls 18.7%.

Several non reported experiments were performed. They suggest that the ability of the model to generate oscillations in the devaluation rate is not very sensitive to parameter values. Hence, this paper successfully explains the first set of empirical regularities mentioned in the Introduction.

Given that the model manages to explain the implementation and collapse of pegging policies, the next question is how well the model manages to explain the business cycle regularities (i.e., the second and third sets of facts) mentioned in the Introduction. The answer to this question requires more care.

In Examples 4 and 5, σ was set equal to zero. The qualitative results would remain the same for any value between 0 and 1. For $\sigma \geq 1$, some qualitative features are not reproduced. For instance, the trade balance x becomes cyclical if $\sigma \geq 1$. For further discussion on the connection between preferences and business cycles regularities in a small open economy context, the reader should consult Correia et al. (1995).

There is an intuitive explanation for the link between devaluation and the real side of the economy. As Rebelo (1997) points out, a drop in the devaluation rate and an increase in output will generate an income effect that leads to a consumption boom. The higher demand for tradable goods is partially offset by imports. Thus, the current account deteriorates. Since the higher demand for

non-tradables cannot be matched by imports, these goods become relatively more expensive. Thus, the real exchange rate appreciates. That is, an income effect induced by a combination of high productivity and low government consumption is the driving force behind the findings of Example 4. The opposite occurs when devaluation increases and output falls in Example 5.

The reader may wonder which type of open market operations and exchange interventions the government will need to carry out to implement the optimal policy. In other words, one may wish to assess the Ramsey values of $B(s^t)$, $B_H^*(s^t)$ and $B_G^*(s^t)$. It is indeed possible to pin down the paths of $B(s^t)$, $B_H^*(s^t)$ and $B_G^*(s^t)$. However, this is true just because I assume that the government issues some specific types of bonds. For instance, if the government is allowed to issue domestic bonds of several maturities, then there will be infinitely many combinations of open market operations and exchange interventions that implement any competitive equilibrium allocation and prices. See Cunha (2013) for further discussion of this problem.

One may ask whether a sufficiently long stretch of bad states could drive the government foreign debt $B_G^*(s^t)$ to some unattainably high value. That cannot happen in a Ramsey equilibrium (at least if S is finite). The reason for that is simple. The optimal values of $B(s^t)$, $B_H^*(s^t)$ and $B_G^*(s^t)$ depend only on s_t. Hence, even if the economy repeatedly hits some bad state, the foreign debt is bounded.

The assessment of the welfare gains from implementing the optimal policy instead of another one chosen as a benchmark (for instance, a policy that keeps the currency depreciation rate constant over time) is another interesting question. I did not carry out this exercise here. However, Araújo and Cunha (2014) study this problem in a closed economy. They conclude that these gains, as percent of GDP, are small.

5. Conclusion

Governments often choose to pursue exchange rate policies that are later abandoned. To understand the driving forces behind the selection of these policies, I study in this paper the problem of choosing an optimal monetary policy with commitment in a context of exogenous fiscal policy. The main finding is that the optimal devaluation rate is correlated in a positive way to the fiscal deficit and in a negative way to technological shocks.

The features of the optimal devaluation policy have a simple explanation. In times of high public expenditures, a benevolent government would like to increase tax rates. If they are exogenous, the only remaining way to raise additional tax revenue is through inflation tax. As the inflation rises, so does the devaluation rate. A negative technological shock will lead to a fall in output. Thus, the ratio between fiscal deficit and output will rise. Again, the government's willingness to raise tax rates explains why the devaluation is higher when there is a bad technology draw.

A currency crisis is often followed by a drop below the trend of consumption and output and a real exchange rate depreciation. When a country pegs the exchange rate, the opposite facts plus a current account deterioration usually take place. Ideally, a model aimed at explaining the implementation and collapse of exchange rate regimes should reproduce these stylized facts. This essay also succeeds in replicating that set of empirical regularities.

The notion that currency crises trigger recessions is widely accepted. In this essay, neither does devaluation cause a slowdown nor does pegging cause a boom. The optimal devaluation rate reacts to technological shocks that hit the economy. If a low productivity shock leads to a recession, to prevent or postpone the devaluation is not an efficient policy.

Most (if not all) of the essays on currency devaluation take exchange rate pegging as given. However, these papers do not try to explain why the exchange rate was ever pegged. This paper adopts a single framework to explain simultaneously both currency crises and peggings. The model can account for successive shifts between periods of low and high devaluation rates. Related papers account for only one devaluation episode.

This paper extends the research line started by Lucas and Stokey (1983) to an open economy. This allows the discussion of the optimal exchange rate policy to go beyond the usual discussion of "pegging versus floating". Between those two policies, there are uncountable others. There is no reason to restrain the discussion only to these two extreme options.

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Appendix

Examples' Solutions

In each example, the lifetime utility (4) is maximized subject to the constraints (1) and (8)-(12). Since the solution also satisfies (13), (14), and (15), that solution is a Ramsey allocation. A proper choice for $p^N(s^0)$ ensures that (13) holds. I prove that (14) is satisfied in Example 1. Concerning the other examples, a numerical verification shows that (14) holds. Constraint (15) is surely satisfied because in each example the Ramsey allocation takes only finitely many values.

It is a well known in the Ramsey policy literature that if the public expenditures are high enough, then the government will always be willing to raise all possible lump-sum revenue at date zero. This implies that the date zero cash-in-advance constraint will hold with equality. Otherwise, the money holdings left over would consist of wealth not taxed away through inflation in a lump-sum fashion.

I use the above property in all examples. Assuming that the date zero cash-in-advance constraint hold as equality, the right hand side of (8) can be simplified. Since in all solutions the government indeed uses the inflation tax, the assumption in question is justified.

Example 1. In this environment, consumers face cash-in-advance $p_t c_{1t} \leq M_{t-1}$ and budget $p_t(c_{1t} + c_{2t}) + M_t + q_t B_t \leq p_t l_t + M_{t-1} + B_{t-1}$ constraints. The government budget constraint is $p_t g_t + M_{t-1} + B_{t-1} = M_t + q_t B_t$. A convenient superset of the competitive equilibrium allocations in which the date zero cash-in-advance constraint holds as equality is characterized by $c_{1t} + c_{2t} + g_t = l_t$, $\sum_{t=0}^{\infty} \beta^t \frac{l_t}{1-l_t} = \frac{1+\beta}{1-\beta}$ and $c_{2t} = 1 - l_t$. Plugging the last equation into the other two one obtains $c_{1t} + 2c_{2t} + g_t = 1$ and $\sum_{t=0}^{\infty} \frac{\beta^t}{c_{2t}} = \frac{2+\beta}{1-\beta}$. The period utility function becomes $u = \log c_1 + 2\log c_2$. Thus, the Ramsey problem is to maximize $\sum_{t=0}^{\infty} \beta^t(\log c_{1t} + 2\log c_{2t})$ subject to those two constraints. Let Λ be the Lagrange multiplier for the second one. Given Λ, the date t allocation is fully characterized by the first constraint and

$$\frac{1}{c_{2t}} + \frac{\Lambda}{2(c_{2t})^2} = \frac{1}{c_{1t}} . \tag{16}$$

Thus, the solution is stationary, in the sense that if $g_t = g_k$ then $(c_{1t}, c_{2t}) = (c_{1k}, c_{2k})$. Next I show that $c_{1t} \leq c_{2t}$, which is the equivalent of constraint (14). Note that

$$\Lambda = 2(c_{2t})^2 \left(\frac{c_{2t} - c_{1t}}{c_{1t} c_{2t}} \right) .$$

Thus, if $c_{1t} > c_{2t}$ for some t, then the same holds for all t. So, it is enough to show that $c_{1t} > c_{2t}$ for all t leads to a contradiction. Combining that inequality to $c_{1t} + 2c_{2t} + g_t = 1$ one obtains

$$3c_{2t} < 1 - g_t \Rightarrow \frac{\beta^t}{c_{2t}} > \frac{3\beta^t}{1 - g_t} > 3\beta^t \Rightarrow \sum_{t=0}^{\infty} \frac{\beta^t}{c_{2t}} > 3 \sum_{t=0}^{\infty} \beta^t > \frac{2 + \beta}{1 - \beta} = \sum_{t=0}^{\infty} \frac{\beta^t}{c_{2t}} ,$$

which is a contradiction. This also implies that $\Lambda > 0$. It remains to show that $c_1 > \bar{c}_1$ and $c_2 > \bar{c}_2$. The constraint $c_{1t} + 2c_{2t} + g_t = 1$ implies that at least one of those inequalities hold. From (16),

$$\frac{c_2 - \bar{c}_2}{c_2 \bar{c}_2} \left[1 + \frac{\Lambda}{2} \left(\frac{1}{c_2} + \frac{1}{\bar{c}_2} \right) \right] = \frac{c_1 - \bar{c}_1}{c_1 \bar{c}_1} .$$

Since $\Lambda > 0$, the term inside brackets is positive. Thus, $c_2 - \bar{c}_2$ and $c_1 - \bar{c}_1$ have the same sign, from which it follows that both inequalities $c_1 > \bar{c}_1$ and $c_2 > \bar{c}_2$ hold. Note that l_t is not constant. Hence, a recursive evaluation of $\sum_{t=0}^{\infty} \beta^t \frac{l_t}{1 - l_t}$ shows that people's real assets are not constant. Thus, the optimal policy prescribes public debt issuance.

It was shown, by means of algebraic manipulations, that the solution of Example 1 is stationary. All examples discussed in this paper have the same property. There is an intuitive reason for this. The maximization problem whose solution provides the optimal allocations has no endogenous state variable. So, the solutions turn out to be stationary.

I solved Examples 2 to 5 numerically. I used the well-known Newton's method (see Burden and Faires, 1997). Solutions were computed with a maximum error of 10^{-9}.

Examples 2 and 3. It is possible to address these two examples in a single general framework. The relevant constraints are $c^T(s^t) + x(s^t) = \theta_t^T \sqrt{l^T(s^t)}$, $c_1^N(s^t) + c_2^N(s^t) + g_t^N = \theta_t^N l^N(s^t)$, $l^T(s^t) + l^N(s^t) = l(s^t)$, $\theta_t^T c_2^N(s^t) = 2\theta_t^N c^T(s^t) \sqrt{l^T(s^t)}$, $(1 - \tau_t)\theta_t^N [1 - l(s^t)] = c_2^N(s^t)$,

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \mu(s^t) \frac{l(s^t)}{1 - l(s^t)} = \frac{2 + \beta}{1 - \beta} ,$$

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \mu(s^t) \frac{x(s^t)}{c^T(s^t)} = 0 ,$$

plus

$$\frac{1}{c^T(s^{t-1}, a)} = \frac{1 - \mu_{ab}}{c^T(s^t, a)} + \frac{\mu_{ab}}{c^T(s^t, b)} \quad \text{and} \quad \frac{1}{c^T(s^{t-1}, b)} = \frac{\mu_{ba}}{c^T(s^t, a)} + \frac{1 - \mu_{ba}}{c^T(s^t, b)} . \quad (17)$$

An analysis of the first order conditions shows that the Ramsey problem admits stationary solutions. In this case, the optimal allocation will satisfy $c^T(s^t) = c^T$.

Therefore, there is no loss of generality in replacing (17) by $c^T(s^t) = c^T$ in the formulation of the maximization problem. Thus, the Ramsey problem is to maximize $\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \mu(s^t) \left\{ \log c^T + \log c_1^N(s^t) + \log c_2^N(s^t) + \log[1 - l(s^t)] \right\}$ subject to the constraints listed above. After some manipulation, the first order conditions can be expressed as a 17×17 non linear system of equations.

Examples 4 and 5. In this environment, constraint (10) can be written as $u_T(t+1) = u_T(0)$. Again, the solution is stationary, with the minor detail that the solution for date zero variables is slightly different. The first order conditions can be reduced to a nonlinear system of 25 equations and 25 variables, which can be solved using Newton's method.