Competitive Equilibrium Hyperinflation under Rational Expectations: A Clarification

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1 – Introduction

In our joint paper with Sallum (Barbosa et al (2006), thereafter BCS) we showed that a hyperinflation path would not be a competitive equilibrium outcome if the public deficit to be financed by issuing money were constant. A question that would come up was whether we had disregarded some competitive equilibrium hyperinflationary paths (as well as hyperdeflationary ones) in our argument. The answer is an unequivocal no. In this brief note we provide a detailed explanation of this matter.

In the set-up of our hyperinflation model, we were very careful to avoid some traps that have plagued the literature on hyperinflation. In our economy, hyperinflation is caused by fundamentals, not by bubbles.

The word bubble has been used in the literature with two different meanings. This is not a just semantic question. There are two types of bubbles: i) speculative bubble equilibrium [as in Obstfeld and Rogoff (1983)] and ii) strict sense bubble [as in Tirole (1982)].[1] The speculative bubble is an equilibrium solution of the model. The strict sense bubble is not an equilibrium because it does not fulfill all the conditions required to be a

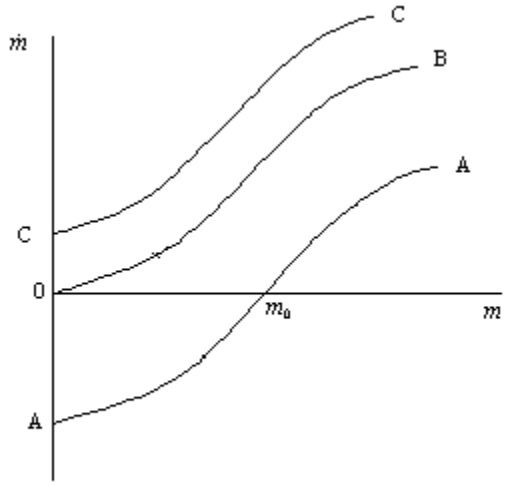
[1] The hyperinflation paths presented, for example, by Blanchard and Fischer [(1989), p.243] and Bruno and Fischer (1990) are strict sense bubbles. It should be pointed out that they did not claim them to be equilibrium solutions of their models.

solution of the model. As we show next, the paths that we disregard were strict sense bubbles.

2 – Hyperinflation Pitfalls

As BCS have done, the use of phase diagram is very helpful to analyzed the case of constant public deficit. The graph below is very similar to BCS's figure 1.

Figure 1



BCS pointed out that the outcome depends on whether the constant fiscal deficit f is greater than, equal to or smaller than $\lim_{m\to 0^+} s(m)$, where the function s measures the value of the services that money provides to the household. So, there are three possible cases to consider:

I: $\lim_{m\to 0^+} s(m) > f$;

II: $\lim_{m\to 0^+} s(m) = f$;

III: $\lim_{m\to 0^+} s(m) < f$.

We discuss each of these cases in turn. In the first case, which corresponds to the locus *AA*, there is a unique steady state m_0. There seem to exist some competitive equilibrium hyperinflation paths starting at the left of m_0. However, these paths do not respect the government's intertemporal budget constraint. The argument here involves some subtlety that has not been appreciated in the literature.[2]

Consider the government budget constraint

$$\dot{m}(t) - \rho m(t) = -\left[s(m(t)) - f\right], \tag{1}$$

which is equivalent to equation (12) on page 187 of BCS. From (1) we obtain

$$m(t)e^{-\rho t} = m(k)e^{-\rho k} + \int_t^k e^{-\rho\theta}\left[s(m(\theta)) - f\right]d\theta \ .$$

Now, recall that in BCS the household consumption c is constant. Therefore, this agent transversality condition $e^{-t\rho}u'(c)m(t) \to 0$ is equivalent to

$$\lim_{k\to\infty} e^{-\rho k} m(k) = 0 \ . \tag{2}$$

This leads to the government's intertemporal budget constraint:

$$m(t) = \int_t^\infty e^{-\rho(\theta - t)}\left[s(m(\theta)) - f\right]d\theta \ . \tag{3}$$

Since the money demand is inelastic, $s'(m) \le 0$. We use this fact and the limit in **I** to conclude that for some sufficiently small (but positive) ε, there exists a value $m(\varepsilon)$ such that

$$m < m(\varepsilon) \Rightarrow s(m) > f + \varepsilon \, .$$

Along a hyperinflation path, m decreases to zero. Hence, for some sufficiently large $t(\varepsilon)$

$$t > t(\varepsilon) \Rightarrow m(t) < m(\varepsilon) \Rightarrow s(m(t)) - f > \varepsilon \, .$$

[2] Barbosa (2002) presents a detailed analysis of this issue.

Combine the last inequality with equation (3) to obtain, for $t > t(\varepsilon)$,

$$m(t) > \varepsilon \int_t^{\infty} e^{-\rho(\theta - t)} d\theta = \frac{\varepsilon}{\rho} > 0 \Rightarrow \liminf_{t \to \infty} m(t) \geq \frac{\varepsilon}{\rho} > 0 \ .$$

However, the last two inequalities contradict the assumption that m decreases to zero. Hence, any hyperinflation path starting to the left of m_0 is not a competitive equilibrium. We should emphasize that this last conclusion has long been neglected in the literature.

Let us now consider case **II**, which is associated with locus $0B$. As BCS mentioned, there is a competitive equilibrium hyperinflation steady state at the origin. There are hyperdeflation paths to the right of the origin. None of them constitute a competitive equilibrium outcome. As we establish this fact, we will simultaneously show that the same holds for the hyperdeflation paths associated with locus CC (which corresponds to case **III**).

The government budget constraint (1) is equivalent to

$$\frac{\dot{m}(t)}{m(t)} = \rho + \frac{f - s\big(m(t)\big)}{m(t)} \ .$$

Recall that $s'(m) \leq 0$. Hence, given the equality in **II** and the inequality in **III**, we can conclude that $f \geq s(m)$ for all values of m. Therefore, we have

$$\frac{\dot{m}(t)}{m(t)} \geq \rho \ .$$

This last inequality is not consistent with the transversality condition (2). As a consequence, a hyperdeflation cannot be a competitive equilibrium outcome.

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