

UFRJ
Matemática Financeira
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Lista 11
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1.

Figura 1

[resolução algébrica] $i = 15,8292\% \text{ a.m.}$

2.

Figura 2

[resolução numérica] $i = 1,9434\% \text{ a.m.}$

3.

Figura 3

$$v_{\min} = \frac{1}{(1 + 0,01)^5} \left[1.200 \frac{1 - (1 + 0,01)^{-4}}{0,01} \right] \implies v_{\min} = 4.455,10$$

4.

Figuras 4a e 4b

V_a : valor atual do fluxo de caixa na Figura 4a

V_b : valor atual do fluxo de caixa na Figura 4b

$$\begin{aligned} V_a &= 10.000 \frac{1 - 1,007^{-36}}{0,007} \\ V_b &= 130.000 + 130.000 \frac{1 - (1,007^6)^{-5}}{1,007^6 - 1} \\ V_a + V_b &= 1.021.552,61 \end{aligned}$$

Como $V_a + V_b > 1.000.000$, a compra deve ser feita à vista.

5.

Figura 5

$$750.000 = X \frac{1 - 1,01^{-24}}{0,01} \implies X = 35.305,10$$

6.

Figura 6

$$10.000,009^2 = X \frac{1 - 1,009^{-6}}{0,009} \implies X = 1.750,65$$

7.

Figura 7

$$C = X \frac{1 - (1+i)^{-n}}{i} \implies \dots \implies n = \frac{\ln\left(\frac{X}{X-iC}\right)}{\ln(1+i)} \implies n = \frac{\ln(5/3)}{\ln(1,08)} \implies n \cong 6,64$$

Conclusão: $n = 6$.

$$20.000 = 4.000 \frac{1 - 1,08^{-6}}{0,08} + \frac{z}{1,08^7} \implies z = 2.585,27$$

Resposta final: haverá 7 prestações, sendo as 6 primeiras de R\$ 4.000,00 e a última de R\$ 2.585,27.

8.

Figura 8

$$C = 3.586,42 \times 1,02^4$$

$$\begin{aligned} C &= X \frac{1 - (1+i)^{-n}}{i} \implies \dots \implies n = \frac{\ln\left(\frac{X}{X-iC}\right)}{\ln(1+i)} \\ \implies n &= \frac{\ln\left(\frac{500}{500 - 0,02 \times 3.586,42 \times 1,02^4}\right)}{\ln(1,02)} \implies n \cong 8,52 \end{aligned}$$

Conclusão: $n = 8$.

$$3.586,42 \times 1,02^4 = 500 \frac{1 - 1,02^{-8}}{0,02} + \frac{z'}{1,02^8} \implies z' = 256,96$$

Resposta final: haverá 8 prestações, sendo as 7 primeiras de R\$ 500,00 e a última de R\$ 256,96.

9.

Figura 9

$$C = \frac{X}{i} \implies i = \frac{X}{C} \implies i = \frac{10.000}{5.000.000} \implies i = 0,2\%$$

10. [completa] Inicialmente, observe que $65 - 30 = 35$ e $35 \times 12 = 420$. A figura que se segue ilustra o fluxo de caixa subjacente ao planejamento elaborado por Fulano.

Figura 10

Seja S o montante disponível na data 420. Desta forma, é preciso que

$$iS = 6.000 \implies S = \frac{6.000}{0,003} \implies S = 2.000.000.$$

Denote por C o valor presente de S na data 0. Logo,

$$C = S \times 1,003^{-420} \implies C = 2.000.000 \times 1,003^{-420}.$$

Conclui-se então que X deve ser tal que

$$X + X \frac{1 - 1,003^{-420}}{0,003} = C.$$

Assim sendo,

$$X \left[1 + \frac{1 - 1,003^{-420}}{0,003} \right] = 2.000.000 \times 1,003^{-420} \implies X = 2.372,17.$$

Consequentemente, Fulano deve fazer aportes mensais de R\$ 2.372,17.

$$11. \quad 98 - 30 = 68 \quad 68 \times 12 = 816 \quad 816 - 420 = 396$$

Figura 11

$$S = 6.000 \frac{1 - 1,003^{-396}}{0,003}$$

$$C = S \times 1,003^{-420} \implies C = 6.000 \frac{1 - 1,003^{-396}}{0,003} \times 1,003^{-420}$$

$$X + X \frac{1 - 1,003^{-420}}{0,003} = C \implies X = \frac{6.000 \frac{1 - 1,003^{-396}}{0,003} \times 1,003^{-420}}{1 + \frac{1 - 1,003^{-420}}{0,003}} \implies X = 1.647,77$$

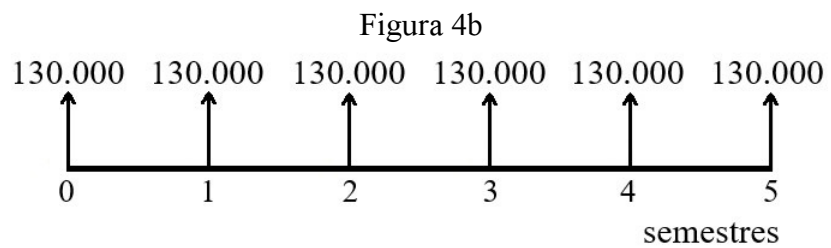
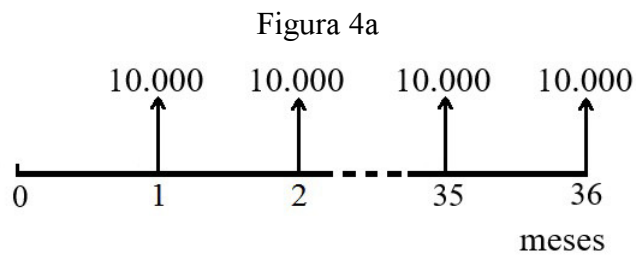
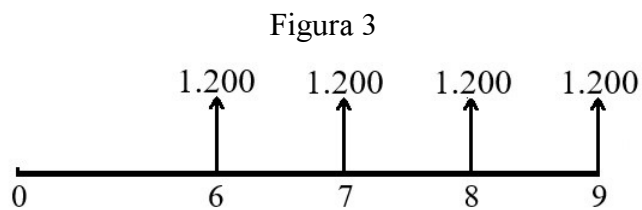
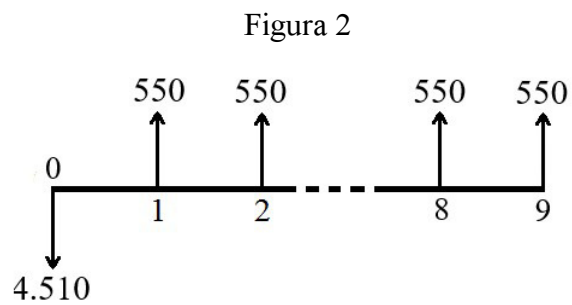
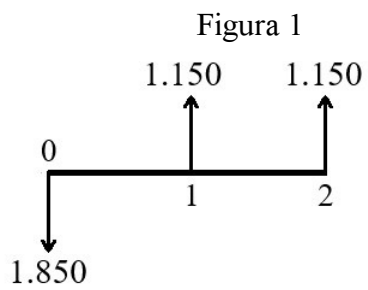


Figura 5

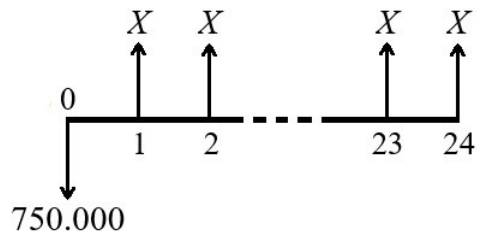


Figura 6

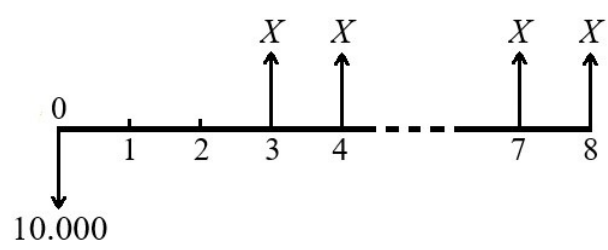


Figura 7

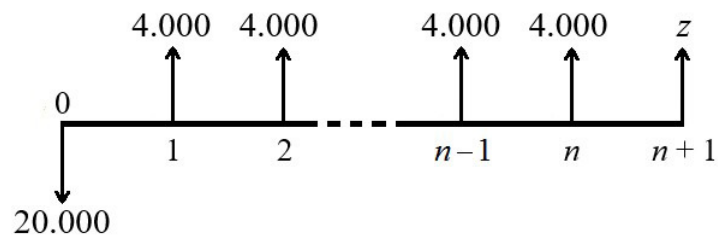


Figura 8

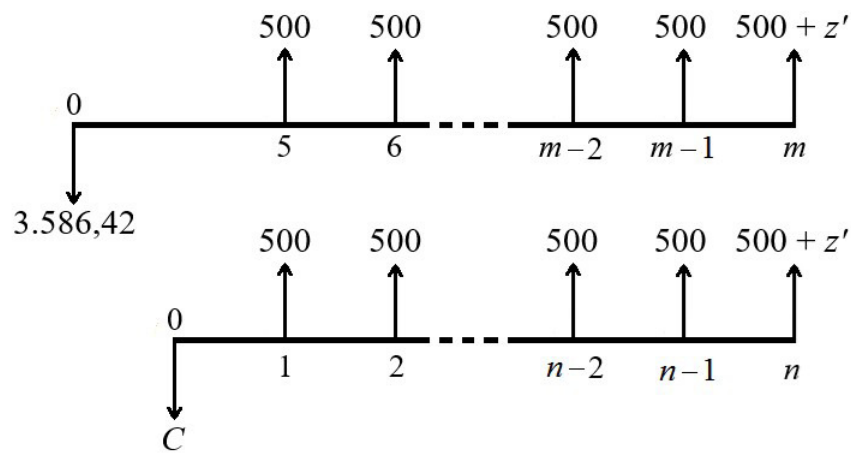


Figura 9

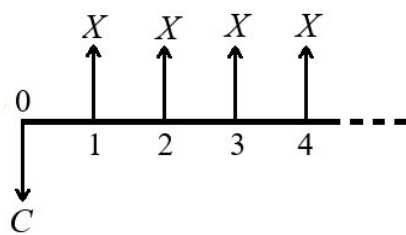


Figura 10

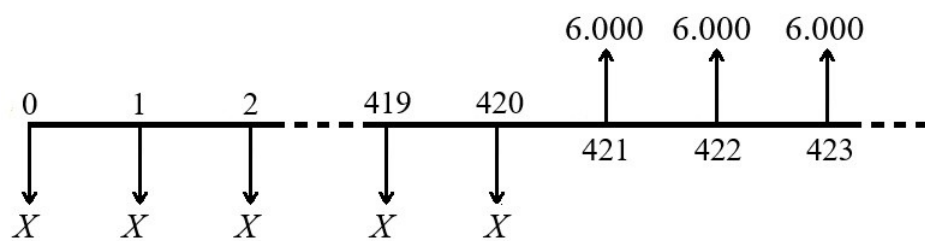


Figura 11

