The Welfare Costs of Business Cycle Smoothing*†

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Abstract: We investigate the welfare implications of macroeconomic policies that eliminate output volatility. We conclude that such policies can lead to welfare losses, even when compared with a simple macroeconomic policy that prescribes constant rates for taxes and money supply growth. We quantify the losses in terms of percentage points of total consumption. We find that they are small, as often found in the related literature.

Keywords: optimal policy, business cycles, welfare.

JEL classification: E32, E61.

1 Introduction

Lucas [7] carried out an exercise that, although extremely simple, had profound impacts on the macroeconomic literature. He measured the costs of business cycles and concluded that they were smaller than 1% of US GDP. Such a finding strongly deviates from the conventional wisdom, which attributes large welfare losses to output and other macroeconomic variables' volatility.

Lucas' exercise motivated several other papers that tried to quantify, in different models, the cost of business cycles. Krusell and Smith [5] and Otrok [10] are typical and relatively recent examples of papers discussing the issue.

In apparently unrelated paper, Chari, Christiano and Kehoe [3] studied the problem of selecting optimal tax rates in a real business cycle model. Among

†The authors acknowledge financial support from the Brazilian Council of Science and Technology (CNPq). This paper is available online at http://professores.ibmecrj.br/abcunha/research/papers/paper15.htm.

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some other findings, these authors concluded that if the government selects policies in an efficient fashion, the economy will still face business cycle fluctuations.

A corollary of the results presented in Chari, Christiano and Kehoe [3] is that if a government pursues a policy that eliminates the business cycle, that policy will lead to welfare losses when compared with the optimal policy. In fact, Lucas [7, page 27] himself anticipated this result. According to him "...there is no reason to think that eliminating all consumption variability is either a feasible or desirable objective of policy...". The same argument is presented with more emphasis in Lucas [8].

This paper constitutes an attempt to unify the literature on business cycle costs and optimal macroeconomic policy. We study the properties of optimal and business cycle smoothing macroeconomic policies in a standard cash and credit good model. We ranked the expected utility levels yielded by these and alternative policies. Of course, we found that the optimal policy yields the highest utility and, as in Chari, Christiano and Kehoe [3] and [2], output and other macro variables are not constant. So, fully smoothing out the business cycle is not a desirable policy.

What is more surprising is that there exists a simple policy that yields higher utility than the one that fully smooths the GDP out. We found that a macroeconomic policy that prescribes constant rates for taxes on labor income and money supply growth is preferable to an active fiscal policy that induces the output to be constant.

We performed the standard real business cycle approach of evaluating the first and second moments of the time series generated by the model. We found that stabilization of GDP involves a decrease in the average output and an increase in the volatility of other real variables when compared to the outcome induced by an optimal policy.

Besides the traditional real business cycle approach, we carried out a more formal inference exercise. We computed autocorrelation functions and spectral densities for the consumption, GDP and inflation series generated by the model. We verified that the optimal macroeconomic policy delivers a relatively smooth consumption path without imposing the strong requirement that all output volatility has to be eliminated.

This paper is organized as follows. Section 2 describes our model economy. Section 3 discusses the definition and the characterization of the competitive equilibrium set. Section 4 describes the parametrization of the model economy. We carry out policy exercises and evaluate their statistical and welfare implications at Section 5. In Section 6 we propose some extensions for this paper. The Appendix contains some first-order conditions, tables and figures.

2 The economy

Our economy is identical to the monetary model studied in Chari, Christiano and Kehoe [2]. There is a country populated by a single infinitely lived household and a government. The household is composed by a shopper and a worker, the latter endowed with L units of time.

The economy produces a single good, which is consumed by the household (c) and by the government (g). Technology is described by $0 \leq y \leq \theta l$, where y is the output of that good and l is the amount of time allocated to work.

Transactions take place in this economy in a particular way. At a first stage of each date t, spot markets for goods and labor services operate. At the second stage, security and currency markets operate.[1]

A currency M circulates in this economy. Securities of a particular type are traded: claims B, with maturity of one period, to one unit of M.

Shoppers face a cash-in-advance constraint. A fraction (c_1) of the purchases of the consumption good must be paid for in cash. Except for the purchases of that good, all other transactions are settled during the security and currency trading session. Therefore, the difference $c - c_1$, denoted by c_2, does not need to be purchased in cash.

Let s_t denote the vector (θ_t, g_t). The sequence $\{s_t\}_{t=0}^{\infty}$ is a Markov process. Its state space is $S = \Theta \times G$, where $\Theta = \{\theta_L, \theta_H\}$, $G = \{g_L, g_H\}$, $\theta_L < \theta_H$ and $g_L < g_H$. The random variables θ_t and g_t are independent. The transition probabilities are $\mu(g_{t+1} = g_t) = \mu_g$ and $\mu(\theta_{t+1} = \theta_t) = \mu_\theta$. For a given s^t in S^t, $\mu(s^t)$ denotes the probability that the first t realizations of the process will be equal to s^t. The realization of s_t is known at the beginning of date t.

Each good is produced by a single competitive firm. Let $l(s^t)$ denote the amount of labor supplied by the household at date t if the history s^t occurs. Other variables indexed by s^t have analogous meanings. Feasibility requires

$$c_1(s^t) + c_2(s^t) + g_t = \theta_t l(s^t) \ . \tag{1}$$

The government finances $\{g_t\}_{t=0}^{\infty}$ issuing and withdrawing M; issuing and redeeming B; and taxing labor income at rate $\tau(s^t)$. Its budget constraint is

$$p(s^t)g_t + B(s^{t-1}) + M(s^{t-1}) = \tau(s^t)w(s^t)l(s^t) + q(s^t)B(s^t) + M(s^t) \ , \tag{2}$$

where $p(s^t)$, $w(s^t)$ and $q(s^t)$ are the respective date t monetary prices of the consumption good, labor services and the security; $M(s^t)$ and $B(s^t)$ are the amount of domestic currency and public debt held by the household at the end of date t. Those variables are conditional on the history of events. A negative value for $B(s^t)$ means that the government is lending to domestic residents. Initial public debt $\bar{B}$ and nominal balances $\bar{M}$ satisfy $\bar{B} = 0$ and $\bar{M} > 0$.

The function $u : \mathbb{R}_+^2 \times [0, L] \to \mathbb{R} \cup \{-\infty\}$,

$$u(c_1, c_2, L - l) = \frac{\left\{[(1-\nu)c_1^\rho + \nu c_2^\rho]^{\frac{1-\gamma}{\rho}} (L-l)^\gamma\right\}^{1-\sigma} - 1}{1-\sigma} \ , \tag{3}$$

[1] In this world, unexpected inflation does not act as a lump sum tax. Therefore, the problem of selecting an optimal policy will have a well defined solution even if the government has some outstanding debt at date zero. See Nicolini [9], especially Section 3, for further details.

is the typical household period utility function. As usual, $\gamma, \nu \in (0,1)$, $\rho < 1$, and $\sigma \geq 0$. Intertemporal preferences are described by

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \mu(s^t) u \left(c_1(s^t), c_2(s^t), L - l(s^t) \right) \ , \tag{4}$$

where $\beta \in (0,1)$. The date t budget constraint of the household is

$$\begin{aligned} p(s^t)[c_1(s^t) + c_2(s^t)] + q(s^t)B(s^t) + M(s^t) \leq \\ [1 - \tau(s^t)]w(s^t)l(s^t) + B(s^{t-1}) + M(s^{t-1}) \ . \end{aligned} \tag{5}$$

The constraint $|B(s^t)/p(s^t)| \leq A < \infty$ prevents Ponzi games. The household also faces the cash-in-advance constraint

$$p(s^t)c_1(s^t) \leq M(s^{t-1}) \ . \tag{6}$$

The household chooses an object $\{[c_1(s^t), c_2(s^t), l(s^t), M(s^t), B(s^t)]_{s^t \in S^t}\}_{t=0}^{\infty}$ to maximize (4) subject to the constraints (5), (6), and $l(s^t) \leq 1$. Except for $B(s^t)$, all these variables are constrained to be non-negative.

The single firm operating in the economy behaves in a competitive fashion. At every period, it chooses $l(s^t)$ to maximize its period profit $p(s^t)\theta_t l(s^t) - w(s^t)l(s^t)$.

3 Competitive equilibrium

A history contingent date t vector $(\tau(s^t), p(s^t), w(s^t), q(s^t))$ is denoted by $\varphi(s^t)$. Date t history contingent allocations $(c_1(s^t), c_2(s^t), l(s^t))$ and asset holdings $(M(s^t), B(s^t))$ are denoted, respectively, by $\chi(s^t)$ and $\zeta(s^t)$. Additionally, $\varphi = \{[\varphi(s^t)]_{s^t \in S^t}\}_{t=0}^{\infty}$, $\chi = \{[\chi(s^t)]_{s^t \in S^t}\}_{t=0}^{\infty}$ and $\zeta = \{[\zeta(s^t)]_{s^t \in S^t}\}_{t=0}^{\infty}$.

A *competitive equilibrium* is is an object (φ, χ, ζ) satisfying: (i) given φ, (χ, ζ) solves the household problem; (ii) $w(s^t) = p(s^t)\theta_t$; (iii) (1) and (2) hold.

Our next step is to characterize a competitive equilibrium. There are several possibilities to carry out this task. We will proceed in a particular way that is convenient to the purposes of this paper, as we explain below.

One of our goals is to compute the optimal policy and the competitive equilibrium allocation (i.e., the Ramsey allocation) induced by that policy. It is well known that the Ramsey allocation can be characterized by a standard maximization problem, provided that the competitive equilibrium is characterized just in terms of allocations. This leads to our characterization choice, which we provide next.

Let $u(s^t)$, $u_1(s^t)$, $u_2(s^t)$, and $u_3(s^t)$ denote, respectively, the value of u and its partial derivatives $\partial u/\partial c_1$, $\partial u/\partial c_2$, and $\partial u/\partial(L-l)$ evaluated at the point $(c_1(s^t), c_2(s^t), L - l(s^t))$. We denote the sum $u_1(s^t)c_1(s^t) + u_2(s^t)c_2(s^t) - u_3(s^t)l(s^t)$ by $W(s^t)$.

It is a well known fact that in Ramsey policies the government uses distorting taxation only after using all available lump-sum revenues. This implies that the date zero cash-in-advance constraint will hold as equality. Otherwise, the money holdings left over would consist of wealth not taxed away through inflation in a lump-sum fashion. So, in what follows, we will assume without loss of generality that $p(s^0)c_1(s^0) = \bar{M}$.

The constraint

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \mu(s^t) W(s^t) = u_1(s^0)c_1(s^0) \tag{7}$$

consolidates the family of period budget constraints in (5), while

$$u_2(s^t) \leq u_1(s^t) \tag{8}$$

ensures that $q(s^t) \leq 1$, that is, money is dominated in rate of return.[2]

As Chari, Christiano and Kehoe [2] pointed out, if an allocation χ satisfies (1), (7) and (8), then it can be decentralized as a competitive equilibrium. Formally, the following proposition can be established:

Proposition 1 *Suppose that $\bar{M} > 0$. If an object (φ, χ, ζ) satisfies (1), (7) and (8), then (φ, χ, ζ) is a competitive equilibrium. Conversely, if an array χ satisfies (1), (7) and (8), then there exist a policy φ and a portfolio ζ such that (φ, χ, ζ) is a competitive equilibrium.*

This above result is standard in the literature on optimal policy. For this reason we do not present a proof here. We refer the interested reader to Chari and Kehoe [1].

4 Parametrization

We borrow most of the parameter values from Chari, Christiano and Kehoe [2]. As they point out, under these values the model economy will replicate the behavior of the US economy in the period 1985-1989. For the sake of completeness, we outline their procedure in this section.

Chari, Christiano and Kehoe [2] calibrated the Markov process for government consumption so that it matched the mean value of the ratio $\frac{g}{y}$ and the variance and serial correlation of the detrended log of government consumption. They calibrated the Markov chain for the technology shocks to replicate the variance and serial correlation of its actual US counterpart.

With respect to preferences, the discount factor β, the share factor γ and the endowment L are the same as in Christiano and Eichenbaum [4]. A regression of the ratio of real balances to aggregate consumption against nominal interest rate determined the values of ν and ρ. Concerning the risk aversion parameter, we will run the policy exercises in Section 5 for $\sigma = 1$, so that the period utility is logarithmic, and $\sigma = 1.5$, which is the value used in Kydland and Prescott's [6] seminal paper.

The table that follows contains the selected parameter values.

[2] We obtained both (7) and (8) from the household's first-order conditions. We list these conditions at the Appendix.

Table 1
Parameter Values

government consumption	$(g_L, g_H, \mu_g) = (350, 402, 0.95)$
technology	$(\theta_L, \theta_H, \mu_\theta) = (0.974, 1.027, 0.91)$
preferences	$(\beta, \gamma, \rho, \nu, L) = (0.97, 0.8, 0.83, 0.57, 5475)$

5 Policy analysis

In this section we compare the implications of distinct macroeconomic policies for the artificial economy studied in this paper. We carry out the traditional real business cycle analysis of first and second moments and the spectral analysis proposed by Söderlind [11]. We also provide welfare comparison of the alternative policies.

We consider three alternative policy regimes: the Ramsey, a second one that specifies constant rates for tax on labor income and money supply growth, and one that ensures that all output oscillations are totally smoothed out. We now further elaborate on this point.

The *Ramsey equilibrium* corresponds to the competitive equilibrium that yields the highest possible expected lifetime utility. Given the result established in Proposition 1, to compute the Ramsey allocation, it suffices to solve the problem of maximizing (4) subject to (1), (7) and (8). The solution of that problem has no endogenous state variable. Thus, it turns out that, for $t \geq 1$ and each s^t, the optimal choice of $(c_1(s^t), c_2(s^t), l(s^t))$ depends only on the current state (θ_t, g_t). Since (θ_t, g_t) lies in a four element set, the Ramsey allocation is characterized by four vectors (c_1, c_2, l), one for each possible realization of (θ_t, g_t), plus an additional vector corresponding to the date zero allocation. This allowed using Newton's method for non-linear system to compute the optimal allocations.[3]

One of the goals of this papers is to evaluate the welfare implications of a macroeconomic policy that stabilizes output. Of course, such a policy must yield less utility than the Ramsey one. What is not clear is whether there exists some simple policy rule that yields higher utility than the one that fully smooths out output. It turns out that such a policy exists. Consider a very simple policy rule, under which money supply grows at a constant rate of 5.935% per year (which is the average value for the US economy for the period 1985-1989) and the tax rate on labor income is constant at some level that balances the government lifetime budget constraint. We call the competitive equilibrium induced by that simply policy rule *the baseline equilibrium.*

Note that Proposition 1 establishes that any competitive equilibrium must satisfy (1), (7) and (8). Thus, the baseline equilibrium must satisfy them too. Additionally, the constraint that the money supply grows at the rate 5.935%

[3] All equilibria discussed in this paper are stationary, in the sense that the date t allocations depend only on (θ_t, g_t). This feature allowed us to use Newton's method to compute all equilibria.

requires

$$\frac{\beta}{\mu(s^t)u_2(s^t)c_1(s^t)}\sum_{s\in S}\mu(s^t,s)u_1(s^t,s)c_1(s^t,s) = 1.05935\ , \tag{9}$$

for every s^t, while the fact that the tax rate on labor income is constant requires the factor

$$\theta_t \frac{u_3(s^t)}{u_2(s^t)} \tag{10}$$

to be constant. We obtained both (9) and (10) from the household's first-order conditions. One can think of our baseline equilibrium as the competitive equilibrium induced by stable and credible macroeconomic policies that the government announces at date zero and follows from then on.

The *tax smoothing equilibrium* is the competitive equilibrium induced by macroeconomic policies that use the tax rate on labor income as an instrument to keep output constant. As the baseline equilibrium, the tax smoothing equilibrium must respect (1), (7) and (8). Additionally, we require it to satisfy (9), because we want the only deviation from the baseline policy to be on the fiscal side. Thus, the only difference in the characterization of the tax smoothing equilibrium from the baseline one is that condition (10) is replaced by

$$\theta_t l(s^t) = \theta_0 l(s^0) \tag{11}$$

for all possible histories s^t. On can see the tax smoothing equilibrium as the competitive equilibrium induced by an active output stabilization policy.

We carry out the forthcoming analysis for two different values for the preference parameter σ. We provide a detailed analysis for the case in which $\sigma = 1$. Since the results are roughly the same, we only briefly discuss the results for the case in which $\sigma = 1.5$.

5.1 First and second moment analysis

We report next the first and second moment of selected variables of the Ramsey equilibrium. As usual, π denotes the inflation rate.[4]

Table 2

First and Second Moments - Ramsey Equilibrium ($\sigma = 1$)

variable	y	$\frac{M}{p}$	$(c_1 + c_2)$	l	τ (%)	π (%)
mean	7.0027	4.7548	6.5872	7.0013	33.7618	-3.0143
std.dev.	0.0246	0.0283	0.0283	0.0188	0.1060	1.7448

We show in Table 3 the sample first and second moments of the baseline equilibrium.

[4] All reported statistics are based on 500 sample realizations, each of them of size 100, of (θ_t, g_t). We also assume that $s_0 = (\theta_H, g_L)$.

Table 3
First and Second Moments - Baseline Equilibrium ($\sigma = 1$)

variable	y	$\frac{M}{p}$	$(c_1 + c_2)$	l	τ (%)	π (%)
mean	7.0004	4.4594	6.4565	6.9991	33.5546	2.7496
std.dev.	0.0249	0.0275	0.0275	0.0198	0	1.7038

Output mean is slightly higher in the Ramsey than in the baseline equilibrium. The opposite happens with its standard deviation. Ramsey real balances and private consumption are higher and less volatile than in the baseline equilibrium. Ramsey labor is slightly higher on average and less volatile. Inflation is higher and less volatile in the baseline equilibrium.

We report next the simulated first and second moments of the tax smoothing equilibrium.

Table 4
First and Second Moments - Tax Smoothing Equilibrium ($\sigma = 1$)

variable	y	$\frac{M}{p}$	$(c_1 + c_2)$	l	τ (%)	π (%)
mean	6.9990	4.4565	6.5809	6.9977	33.6902	2.7508
std.dev.	0	0.0323	0.0323	0.0252	2.9285	1.5708

Compared to both the Ramsey and baseline equilibria, the tax smoothing equilibrium has a lower average value for output and hours worked. With the exception of output and inflation, all variables are more volatile in the tax smoothing equilibrium than in the baseline.

We observe that the elimination of output volatility has two types of implications. First, it has to be done at the expense of a lower average output. Second, the volatility of other real variables has to rise.

We report in the Appendix the cross-correlation matrices of each experiment. Moreover, we also carried out the first and second moment analysis for $\sigma = 1.5$. The results are virtually the same. Consequently, we do not report them.

5.2 Frequency domain analysis

In the present subsection we will follow Söderlind [11] and perform a more formal inference exercise. Our aim is to compare cyclical properties of the three policy regimes with US data. However, we want to point out that none of the equilibria computed are designed to match statistical properties of macroeconomic variables. The model economy presented in this paper does not have enough structure to capture all relevant cyclical properties. For instance, it has no physical capital and the only existing friction is the cash-in-advance constraint.

Despite the aforementioned limitations, an empirical assessment of policy experiments is a valuable task for three reasons. First, a comparison between data and model results may stress in which dimensions the model is performing worse, which may suggest the incorporation of specific frictions in futures extensions of the model. Second, the empirical assessment may work as a robustness test in the sense that it is able to show how many features of the data a very simple artificial economy is capable of mimicking. Third, the empirical assessment, by showing how time series implications of the equilibria computed are different from the real world, provides us with a counterfactual macroeconomic scenario. In other words, since policy makers are probably not strictly following any of the policy experiments studied here, the cyclical properties generated by the equilibria computed may be thought of as the properties one would expect to find in the data if policy makers were behaving precisely as described by the particular equilibrium.

The approach implemented in this subsection can be thought of as testing the null hypothesis that the time series of US macroeconomic data are plausible realizations of the particular equilibrium studied. We construct 95% confidence intervals for the statistics studied based on 500 replications of length 100 generated by each policy regime.

We focus on three macroeconomic series: detrended GDP, detrended aggregate consumption and inflation. We compute statistics in the time domain and in the frequency domain. In particular, we compute autocorrelation function and spectral density for each time series in order to analyze persistence and volatility in a dynamic way. The autocorrelation functions measure persistence over different horizons and the area under the spectral density is the variance associated with a particular frequency band, which represents cycles of a particular periodicity.

To study the comovement of consumption and inflation with GDP, we evaluate cross-correlation functions and coherence, which is a frequency domain analogue of R^2 of basic regression analysis. It is worth noticing that since in the tax smoothing equilibrium GDP is kept constant by design, there is no point in computing comovement measures or autocorrelation function and spectral density for detrended GDP in that particular equilibrium.

The results presented in the Appendix are based on the parametrization in which $\sigma = 1$. The solid line represents statistics based on US quarterly data from 1978 to 2002. Figures 1, 3 and 5 show a panel containing autocorrelation functions and spectral density for GDP, aggregate consumption and inflation. Figures 2 and 4 display measures of comovement with GDP for aggregate consumption and inflation. Figures 1 and 2 show cyclical properties for the Ramsey equilibrium. Figures 3 and 4 are associated with the baseline equilibrium. Figure 5 displays statistical properties for the tax smoothing equilibrium.

The Ramsey and the baseline equilibria are capable of generating GDP realizations that are persistent and almost as volatile as US detrended GDP. Concerning GDP dynamics, both equilibria behave basically in the same way. Concerning consumption, the Ramsey equilibrium delivers a much less persistent consumption series, which comoves weakly with GDP, especially in the low frequencies range. Consumption in the Ramsey equilibrium is almost independent of GDP. All policy experiments generate inflation realizations that lack persistence and have low volatility. Consumption in the Ramsey and in the tax smoothing equilibrium is less volatile than consumption in the baseline equilibrium.

The Ramsey equilibrium provides a smooth consumption path without requiring constant GDP, since the spectral density generated by the model shows a substantial degree of volatility in GDP. By contrast, the tax smoothing equilibrium displays constant output. Therefore, the Ramsey equilibrium is able to achieve a smooth pattern of consumption without demanding the draconian technological restriction of constant production. This is why the Ramsey equilibrium generates a higher level of welfare than total elimination of output fluctuations.

5.3 Welfare implications

Let U^r, U^b and U^τ denote, respectively, the expected lifetime utility under the Ramsey, baseline and tax smoothing policies. We found out that $U^r > U^b > U^\tau$ for both $\sigma = 1$ and $\sigma = 1.5$. This finding suggests that, contrary to the conventional wisdom, output variability and business cycles are not necessarily costly.

The definition of Ramsey equilibrium implies the inequalities $U^r > U^b$ and $U^r > U^\tau$. It remains to understand why U^τ is smaller than U^b.

A key variable to understand the above utility ranking is the tax rate on labor income. In the baseline equilibrium, τ is constant and approximately equal to 33.55%. The Ramsey tax on labor income is roughly constant, since $33.63\% \simeq \tau(\theta_H, g_L) < \tau(\theta_L, g_L) < \tau(\theta_H, g_H) < \tau(\theta_L, g_H) \simeq 33.93\%$. However, the same does not apply in the tax smoothing equilibrium. The tax rate ranges from 38.29% at state (θ_H, g_H) to 29.12% at state (θ_L, g_L). The requirement of keeping output constant drives the tax rate on labor away from its optimal path. This does not happen in the baseline equilibrium. The cross-correlation matrices in the Appendix also help to illustrate this point.

We also measured the benefits of shifting from the baseline to the Ramsey policies. Following the procedure that is standard in the literature on the costs of business cycle fluctuations, we measured the benefit as the uniform (i.e., over all t and all s^t) percentage point increase in the baseline c_1 and c_2 that would make people indifferent between the baseline and the Ramsey policies. Our findings are shown in Table 5.

Table 5
Welfare Gains and Losses

policy experiment	$\sigma = 1$	$\sigma = 1.5$
from baseline to Ramsey	0.1716%	0.1713%
from baseline to tax smoothing	-0.0930%	-0.0928%

As usual in the related literature, we obtained small figures. However, in contrast to most authors, we found that policies aimed at smoothing out business cycles are not necessarily welfare improving.

6 Future research

We plan to improve this paper in several dimensions. The most important improvement is to introduce physical capital in the model. In the present version, the stabilization of output requires a perfect crowding out of private consumption whenever g hits a relatively high value. In a model with physical capital, investment is likely to bear part of that crowding out.

We mentioned in Section 4 that we based our parametrization on the calibration exercise of Chari, Christiano and Kehoe [2]. These authors calibrated the model for the years 1959-1989. We plan to execute our own calibration for a more recent period.

We also plan to perform several other policy exercises. In this paper, we have only used the tax on labor income as a policy stabilization instrument. We plan to evaluate the use of monetary policy in the same role, as well as mixes of monetary and fiscal policy.

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Appendix

Household's First–Order Conditions

If $\bar{M}$ is positive, the first-order necessary and sufficient conditions for a typical household are

$$\beta^t \mu(s^t) u_1(s^t) = [\lambda(s^t) + \xi(s^t)] p(s^t) \ ; \tag{12}$$

$$\beta^t \mu(s^t) u_2(s^t) = \lambda(s^t) p(s^t) \ ; \tag{13}$$

$$\beta^t \mu(s^t) u_3(s^t) = \lambda(s^t)[1 - \tau(s^t)] w(s^t) \ ; \tag{14}$$

$$\lambda(s^t) = \sum_{s_{t+1} \in S} [\lambda(s^t, s_{t+1}) + \xi(s^t, s_{t+1})] \ ; \tag{15}$$

$$\lambda(s^t) q(s^t) = \sum_{s_{t+1} \in S} \lambda(s^t, s_{t+1}) \ ; \tag{16}$$

$$M(s^{t-1}) \geq p(s^t) c_1(s^t) \ \& \ \xi(s^t)[M(s^{t-1}) - p(s^t) c_1(s^t)] = 0 \ ; \tag{17}$$

$$\begin{aligned} p(s^t)[c_1(s^t) + c_2(s^t)] + q(s^t) B(s^t) + M(s^t) = \\ [1 - \tau(s^t)] w(s^t) l(s^t) + B(s^{t-1}) + M(s^{t-1}) \ ; \end{aligned} \tag{18}$$

$$\lim_{t \to \infty} \sum_{s^t \in S^t} \lambda(s^t) M(s^t) = \lim_{t \to \infty} \sum_{s^t \in S^t} \lambda(s^t) q(s^t) B(s^t) = 0 \ ; \tag{19}$$

$$c_1(s^t), c_2(s^t), l(s^t), M(s^t), \lambda(s^t), \xi(s^t) \geq 0 \ , \ l(s^t) \leq L \ ; \tag{20}$$

where $\lambda(s^t)$ and $\xi(s^t)$ are Lagrange multipliers for, respectively, budget and cash-in-advance constraints.

Cross-Correlation Matrices

Table 6

Ramsey Equilibrium ($\sigma = 1$)

variables	y	$(c_1 + c_2)$	L	M/p	c_2	g	θ	τ
y	1							
$(c_1 + c_2)$	0.5538	1						
l	0.3430	-0.5632	1					
M/p	0.5538	1.0000	-0.5632	1				
c_2	0.5538	1.0000	-0.5632	1.0000	1			
g	0.6615	-0.2259	0.9242	-0.2259	-0.2259	1		
θ	0.7074	0.9790	-0.3906	0.9790	0.9790	-0.0329	1	
τ	0.3435	-0.5628	1.0000	-0.5628	-0.5628	0.9243	-0.3902	
π	-0.2025	-0.3089	0.1537	-0.3089	-0.3089	0.0393	-0.3077	0.1536

Table 7

Baseline Equilibrium ($\sigma = 1$)

variables	y	$(c_1 + c_2)$	L	M/p	c_2	g	θ	τ
y	1							
$(c_1 + c_2)$	0.5560	1						
l	0.3765	-0.5305	1					
M/p	0.5560	1.0000	-0.5305	1				
c_2	0.5560	1.0000	-0.5305	1.0000	1			
g	0.6884	-0.1882	0.9242	-0.1882	-0.1882	1		
θ	0.6809	0.9865	-0.3907	0.9865	0.9865	-0.0329	1	
τ	-	-	-	-	-	-	-	
π	-0.2015	-0.3101	0.1466	-0.3101	-0.3101	0.0309	-0.3097	0.2533

Table 8

Tax Smoothing Equilibrium ($\sigma = 1$)

variables	y	$(c_1 + c_2)$	L	M/p	c_2	g	θ	τ
y	-							
$(c_1 + c_2)$	-	1						
l	-	-0.0329	1					
M/p	-	1.0000	-0.0329	1				
c_2	-	1.0000	-0.0329	1.0000	1			
g	-	-1.0000	0.0329	-1.0000	-1.0000	1		
θ	-	0.0329	-1.0000	0.0329	0.0329	-0.0329	1	
τ	-	-0.6894	-0.6796	-0.6894	-0.6894	0.6894	0.6796	
π	-	-0.2514	-0.0133	-0.2514	-0.2514	0.2514	0.0133	0.1886

FIGURES

Figure 1
Autocorrelation Function and Spectral Density
Ramsey Equilibrium

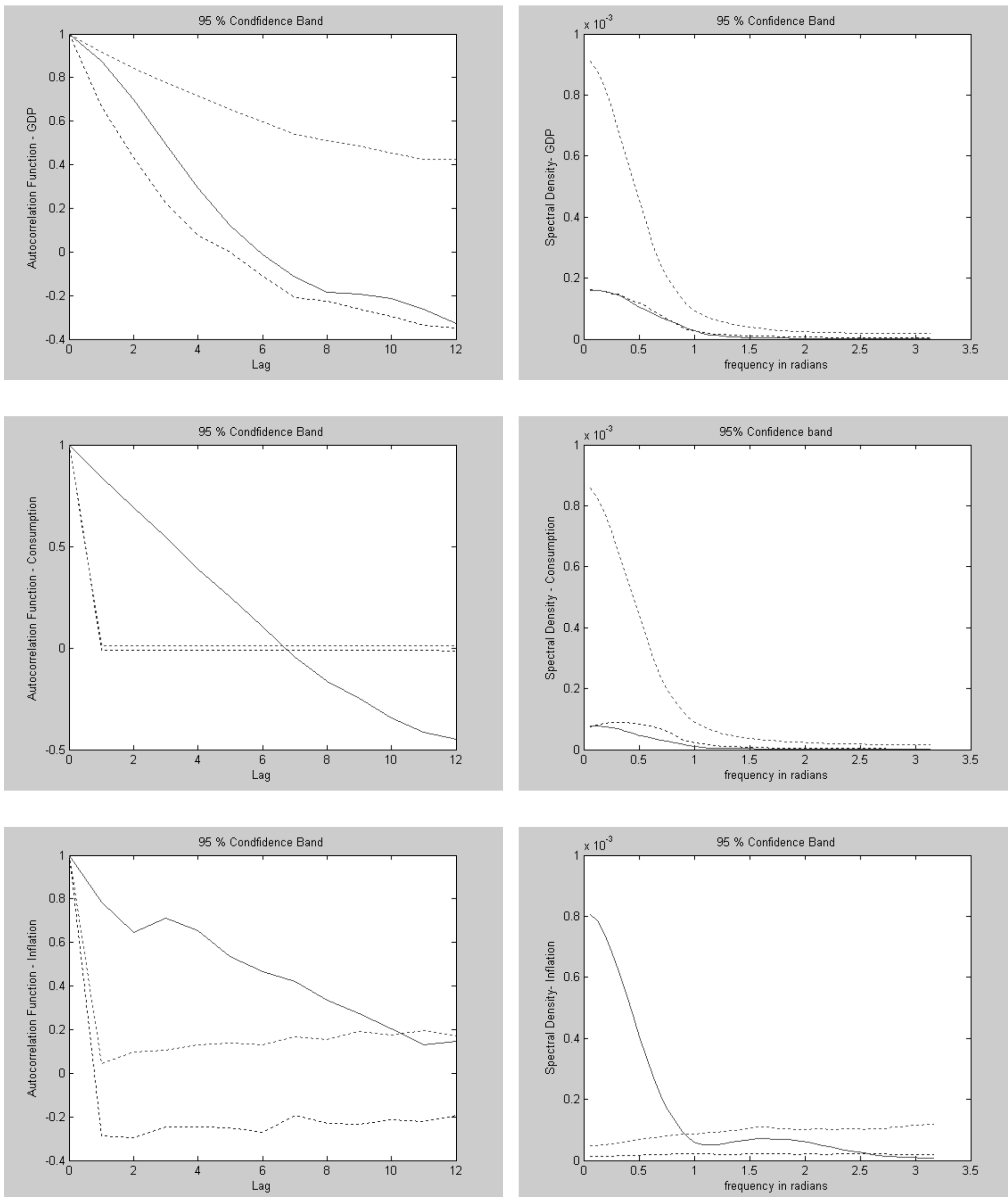

Figure 2
Cross-Correlation Function and Coherence
Ramsey Equilibrium

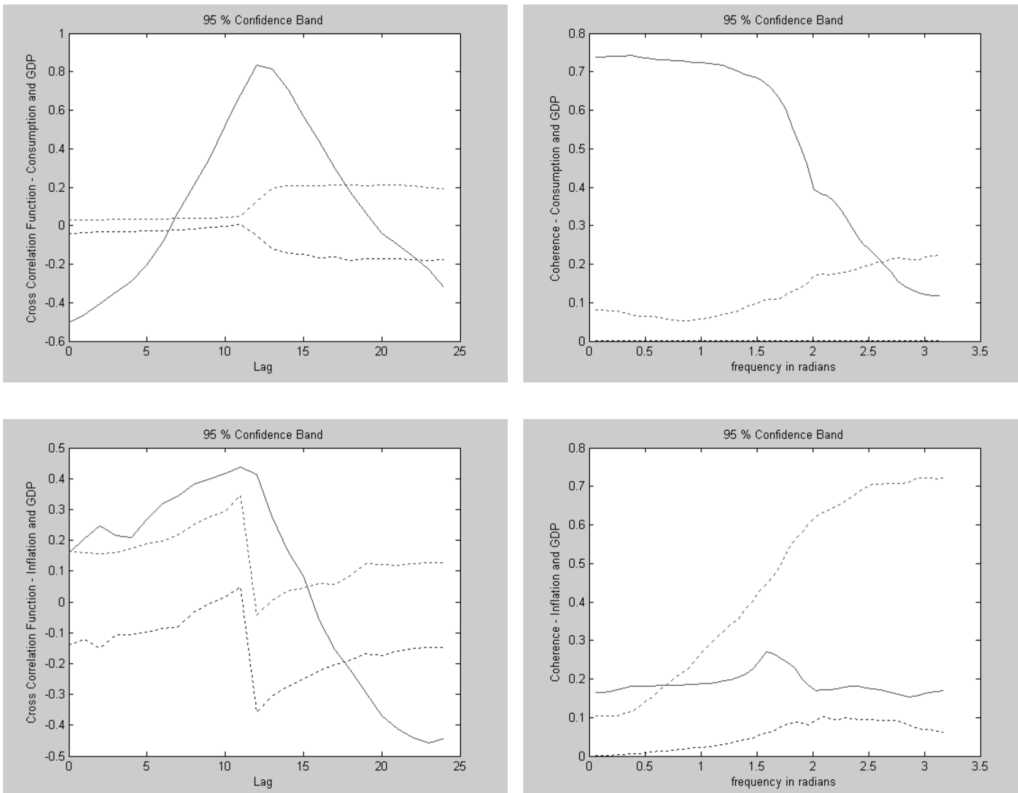

Figure 3
Autocorrelation Function and Spectral Density
Baseline Equilibrium

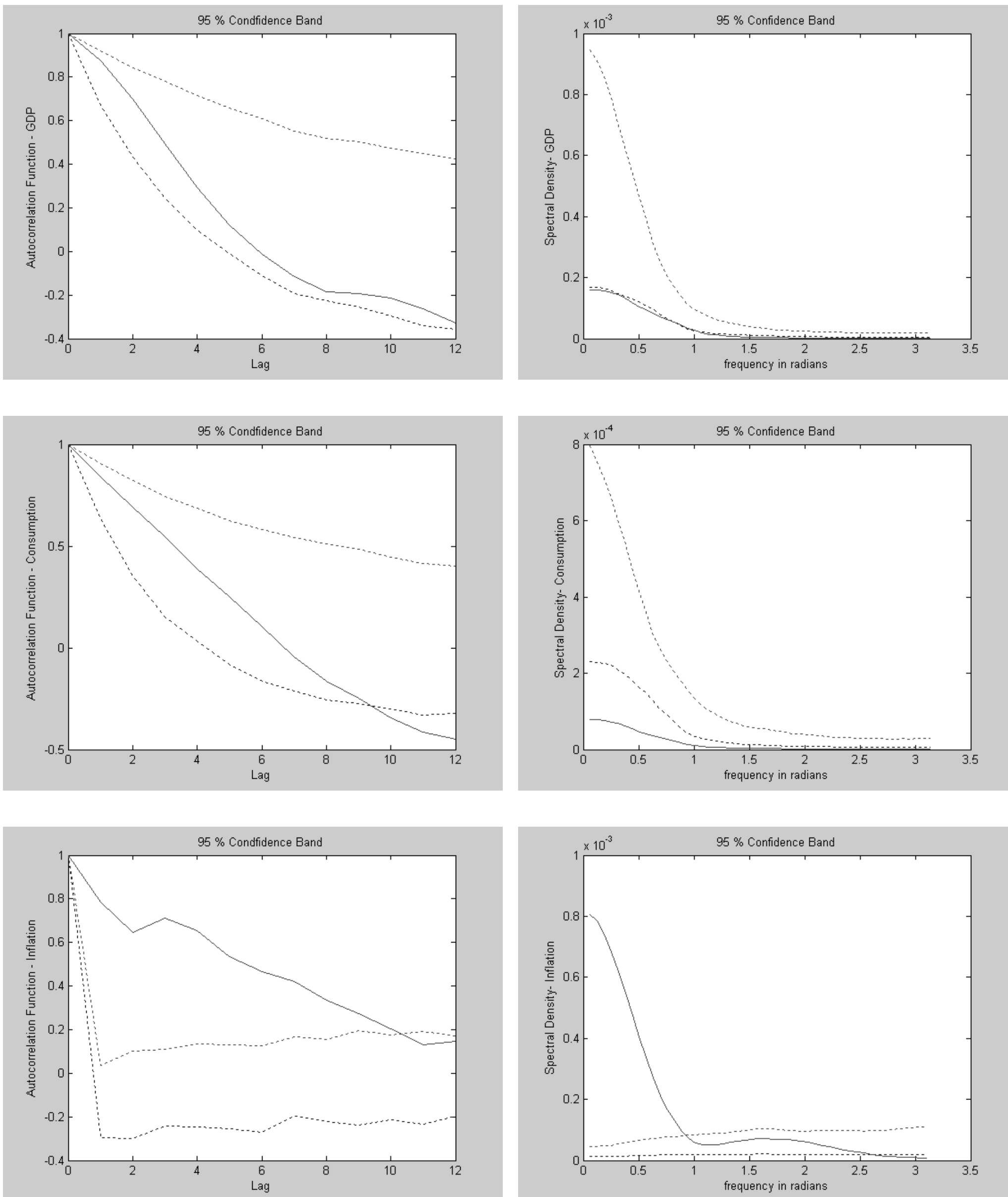

Figure 4
Cross-Correlation Function and Coherence
Baseline Equilibrium

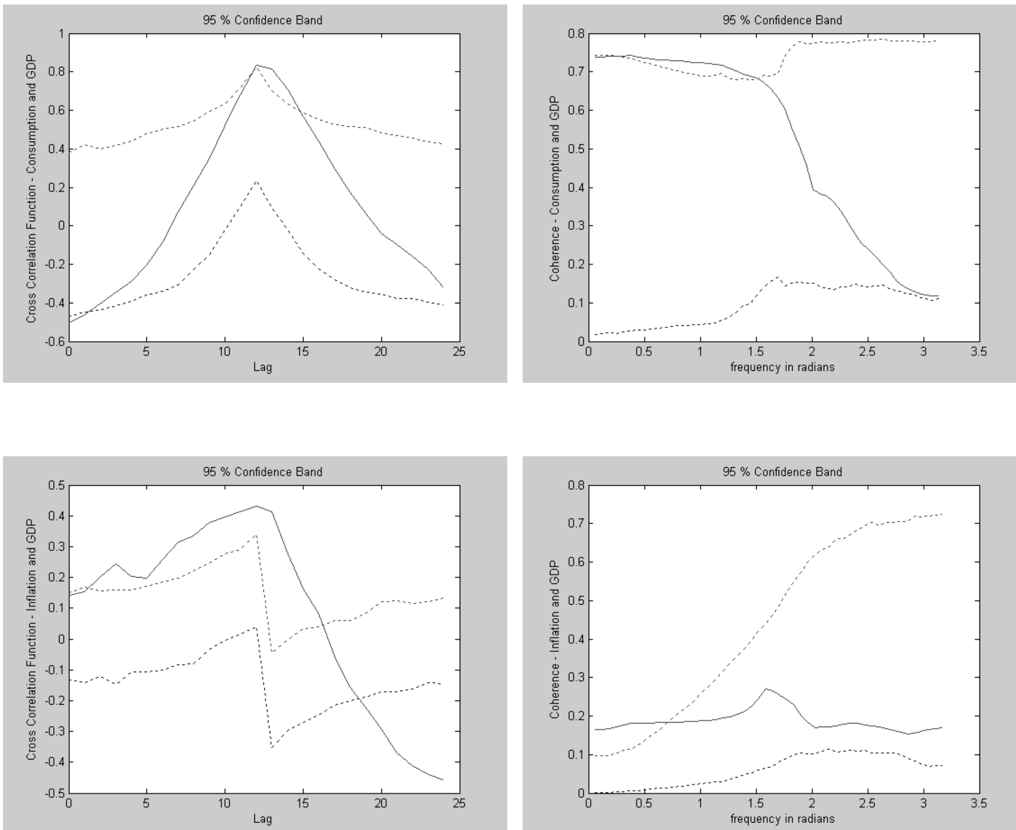

Figure 5
Autocorrelation Function and Spectral Density
Tax Smoothing Equilibrium